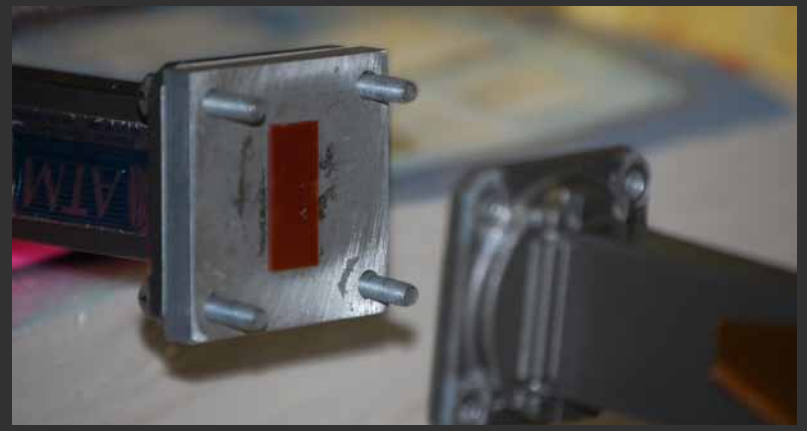
Error estimation in VNA Measurements with applications to permittivity measurements

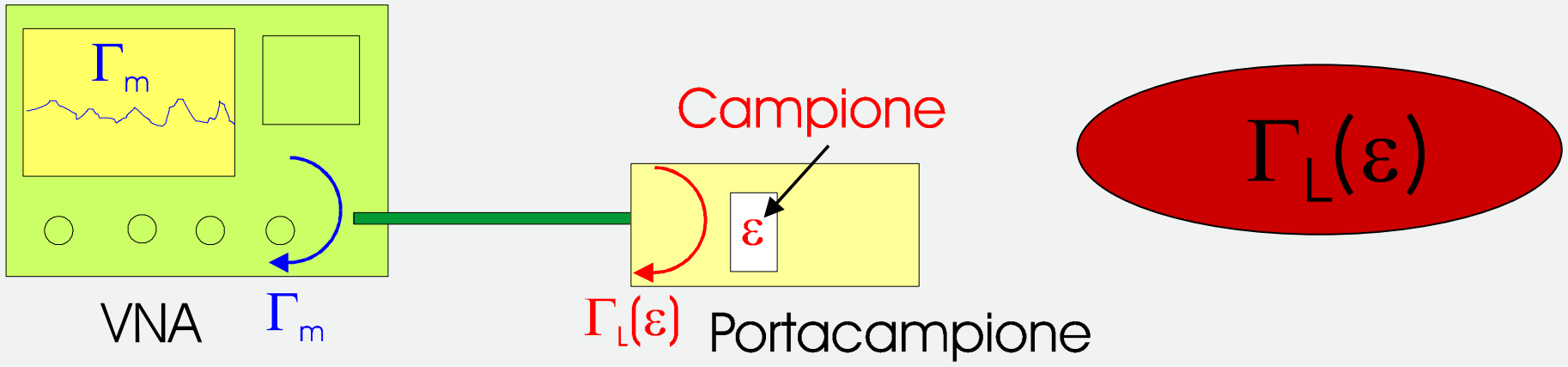
Marco Donald Migliore

University of Cassino and Southern Lazio



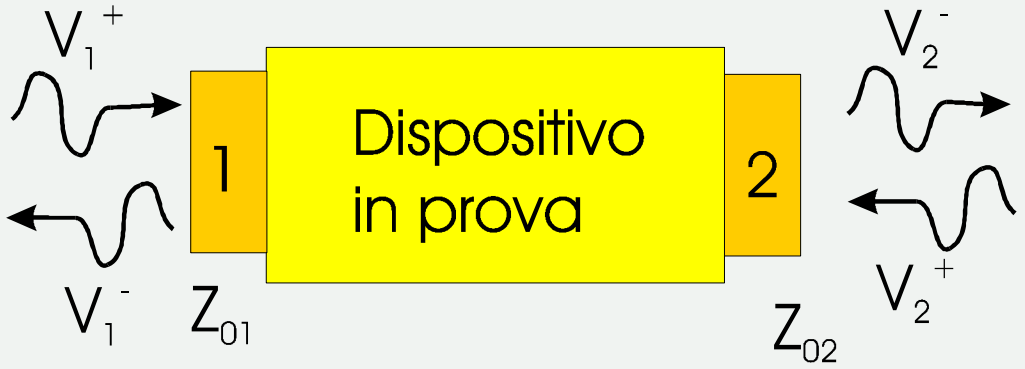
Materials measurement in waveguide

General approach



- Identify the functional relationship between a measured quantity (reflection/transmission coefficient) and the permittivity of the material
- Inversion of the relationship
- It is basically and inverse problem

Network Analyzer



(Generalized) diffusion matrix

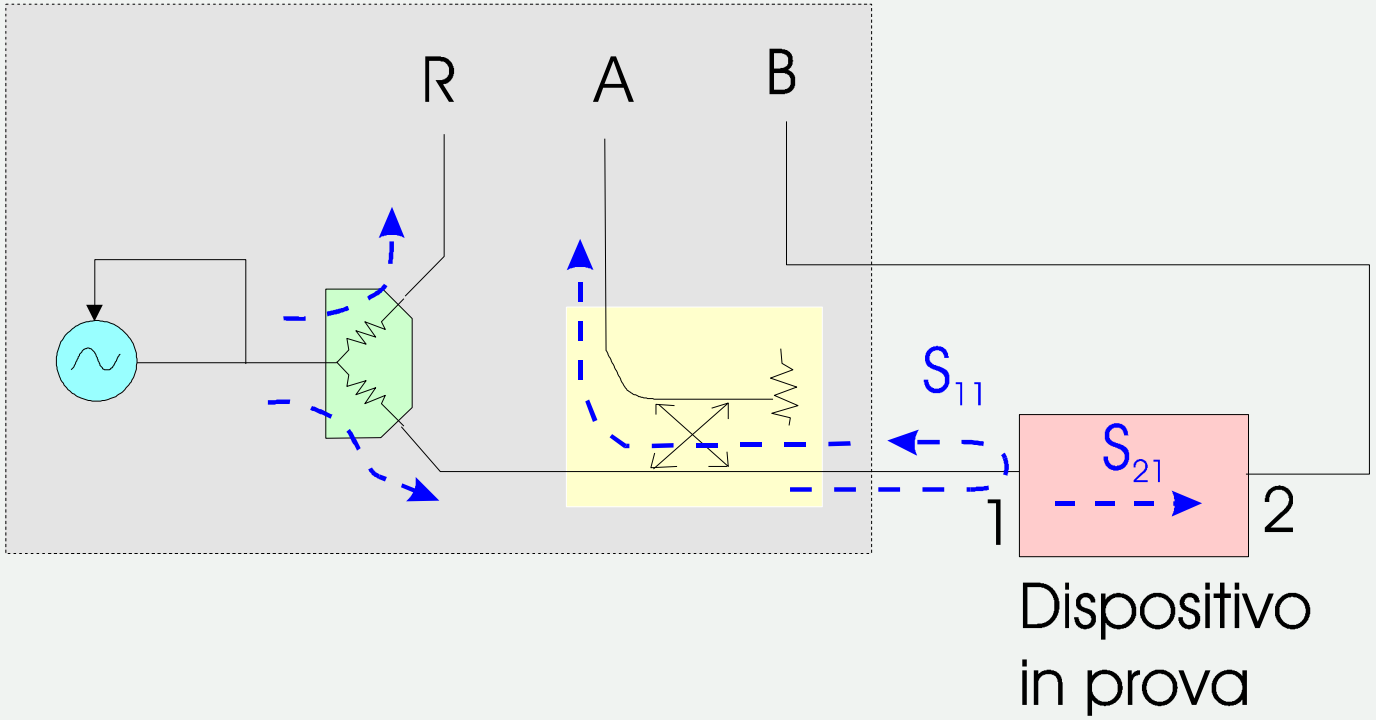
$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$S_{ij} = \left.\frac{b_i}{a_j}\right|_{a_k=0,\, k\neq j} = \left.\frac{V_i^- \sqrt{Z_{0j}}}{V_j^+ \sqrt{Z_{0i}}}\right|_{V_k^+=0,\, k\neq j}$$

$$a_1 = \frac{1}{\sqrt{2}} \frac{V_1^+}{\sqrt{Z_{01}}} \qquad b_1 = \frac{1}{\sqrt{2}} \frac{V_1^-}{\sqrt{Z_{01}}}$$

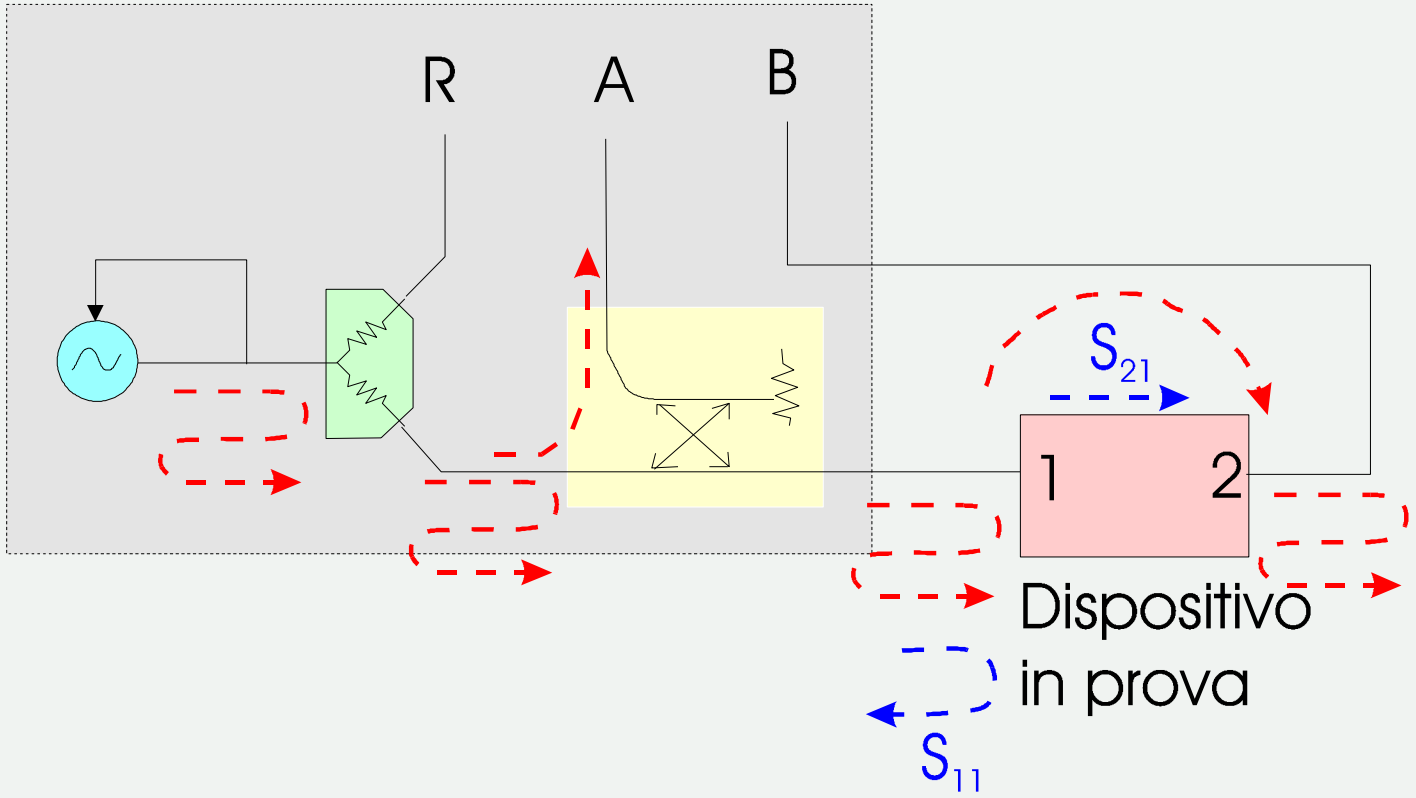
$$a_2 = \frac{1}{\sqrt{2}} \frac{V_2^+}{\sqrt{Z_{02}}} \qquad b_2 = \frac{1}{\sqrt{2}} \frac{V_2^-}{\sqrt{Z_{02}}}$$

An example of an S-parameter measuring circuit

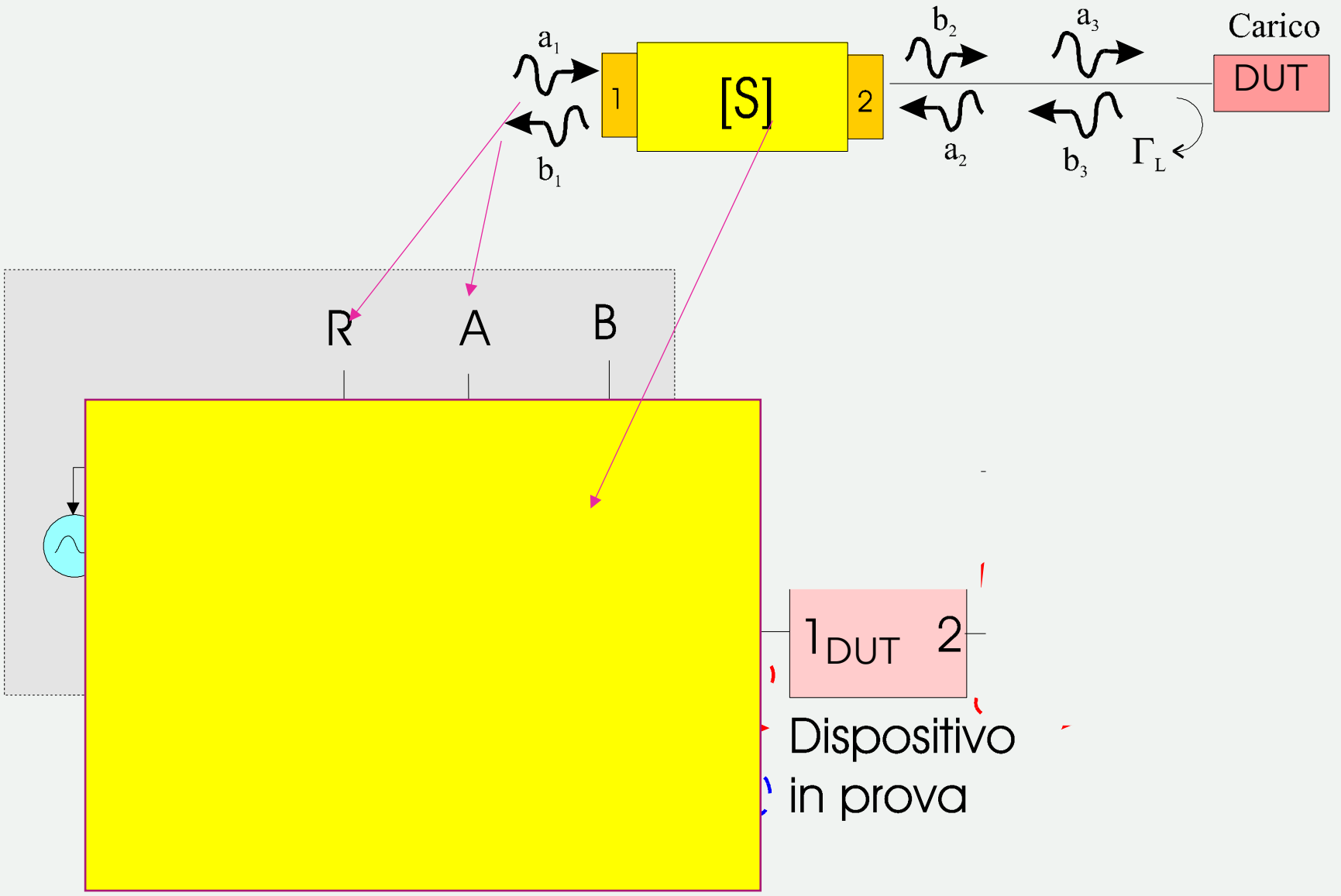


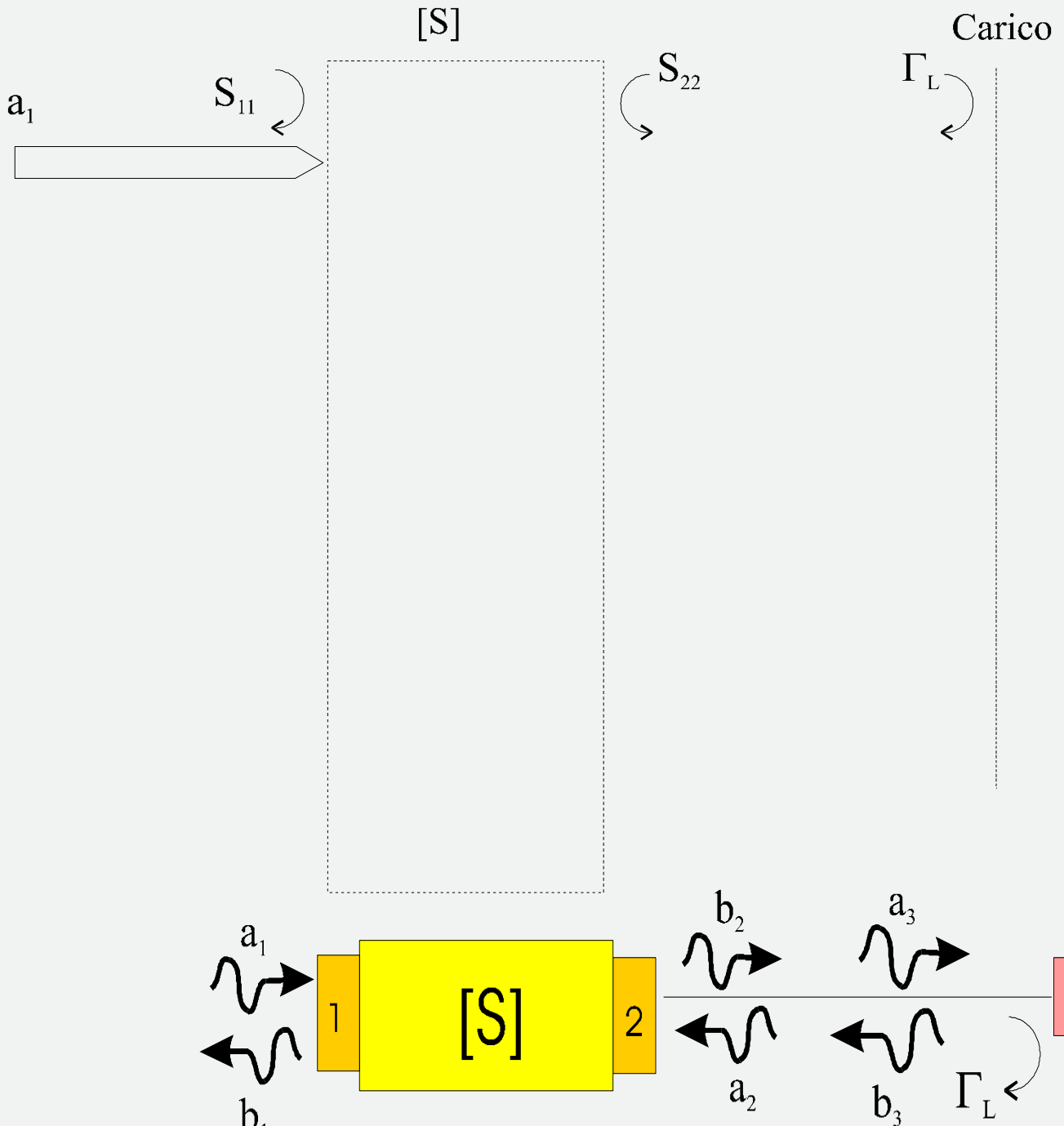
$$\frac{A}{R} = S_{11} \qquad \frac{B}{R} = S_{21}$$

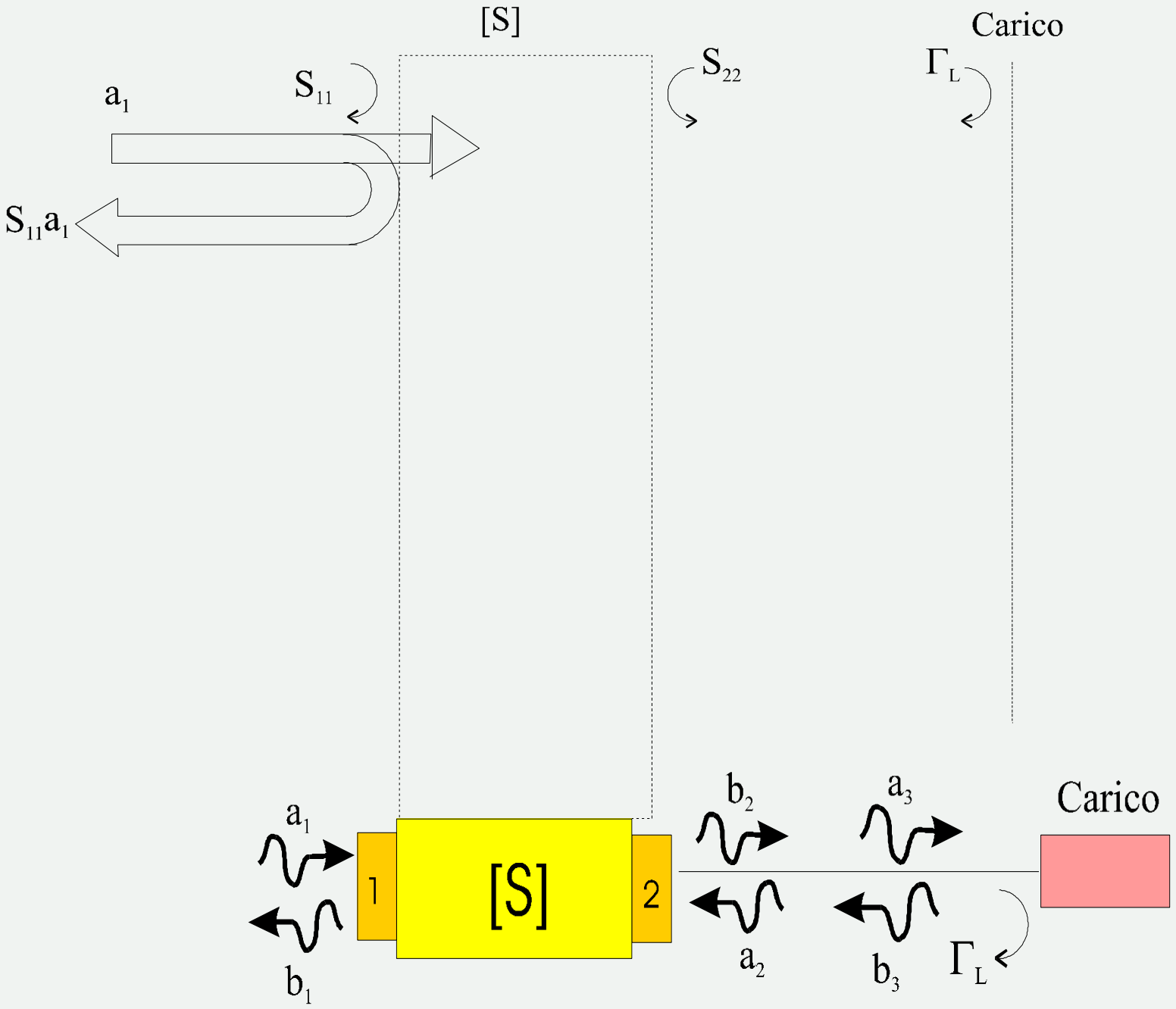
But …

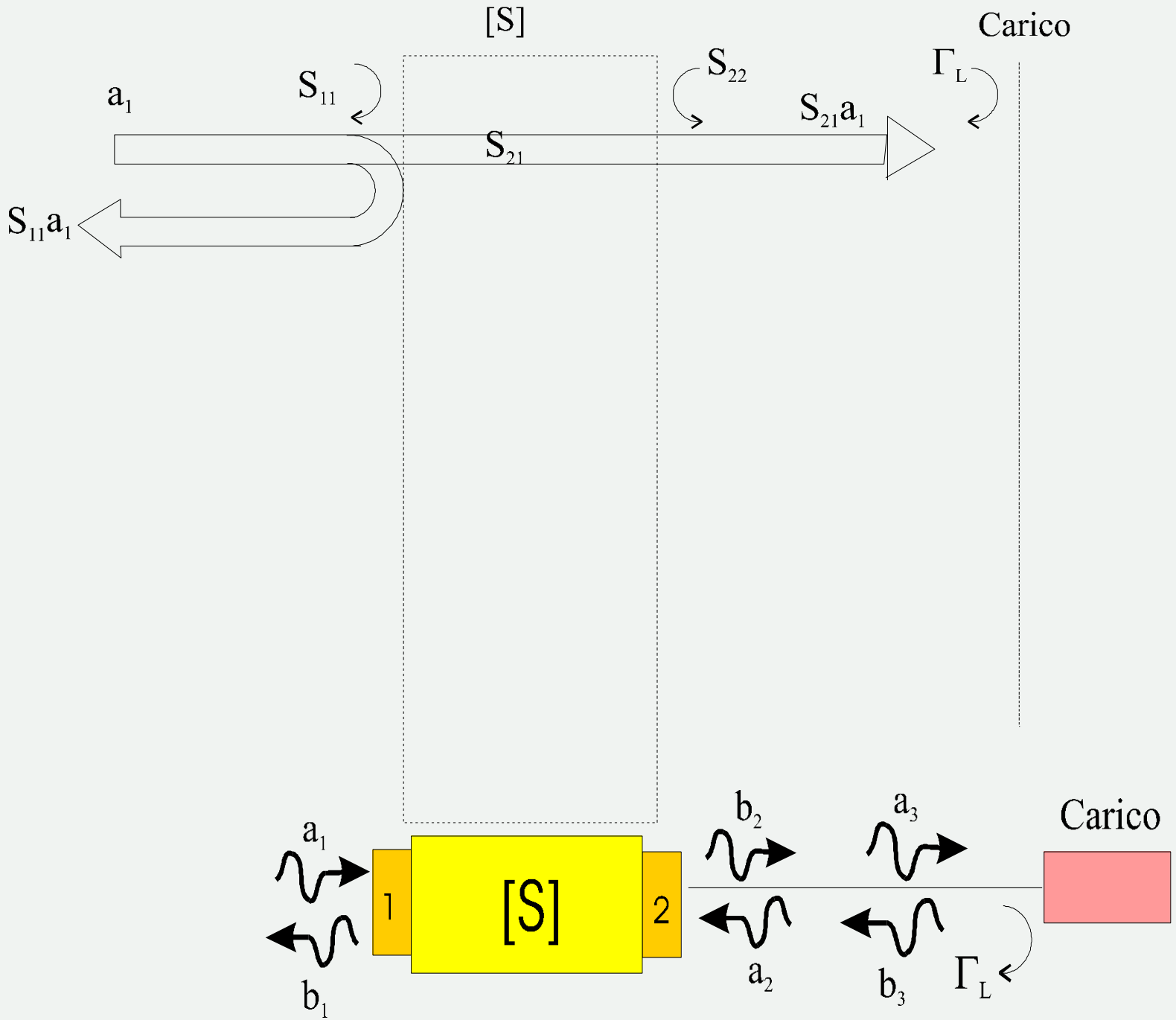


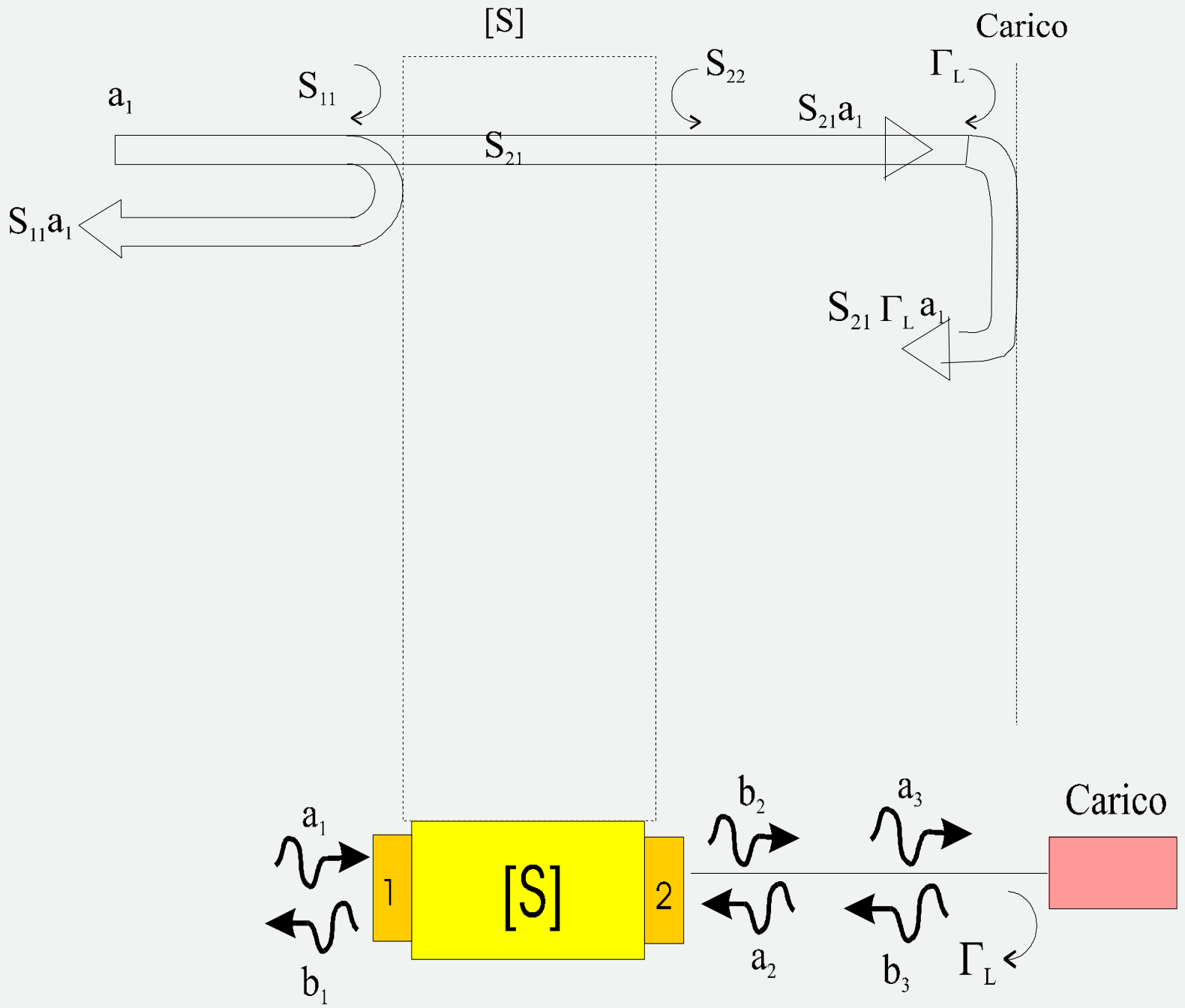
$$\frac{A}{R} \neq S_{11} \qquad \frac{B}{R} \neq S_{21}$$

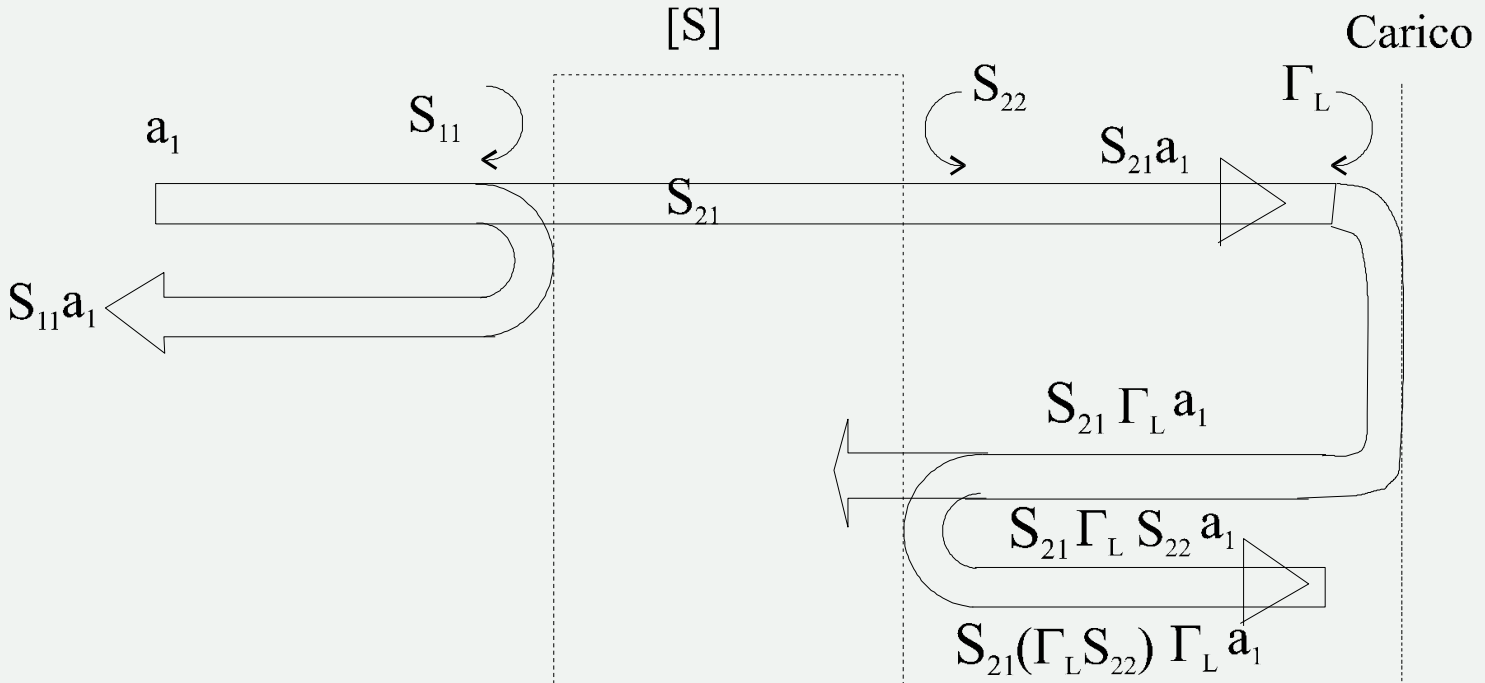

a1
b1
1
[S]
2
b2
a2
a3
b3
ΓL
Carico
DUT
R
A
B
1 DUT 2
Dispositivo
in prova

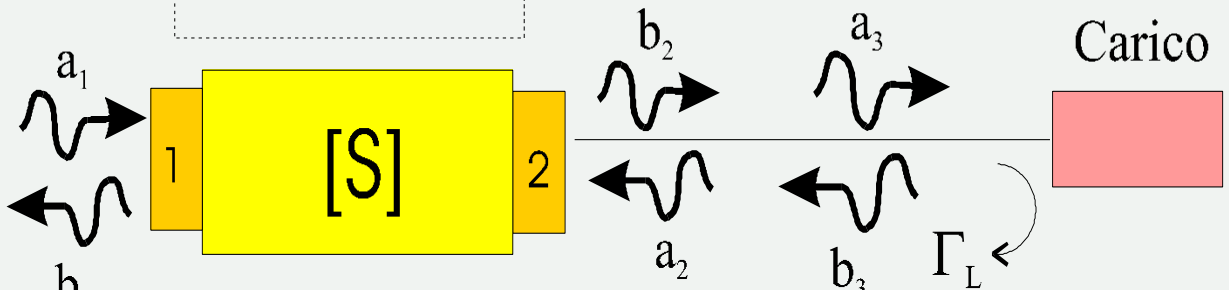

[S]
Carico
a_1
S_{11}
S_{22}
Γ_L
a_1
1
[S]
2
b_2
a_3
Carico
a_2
b_3
Γ_L

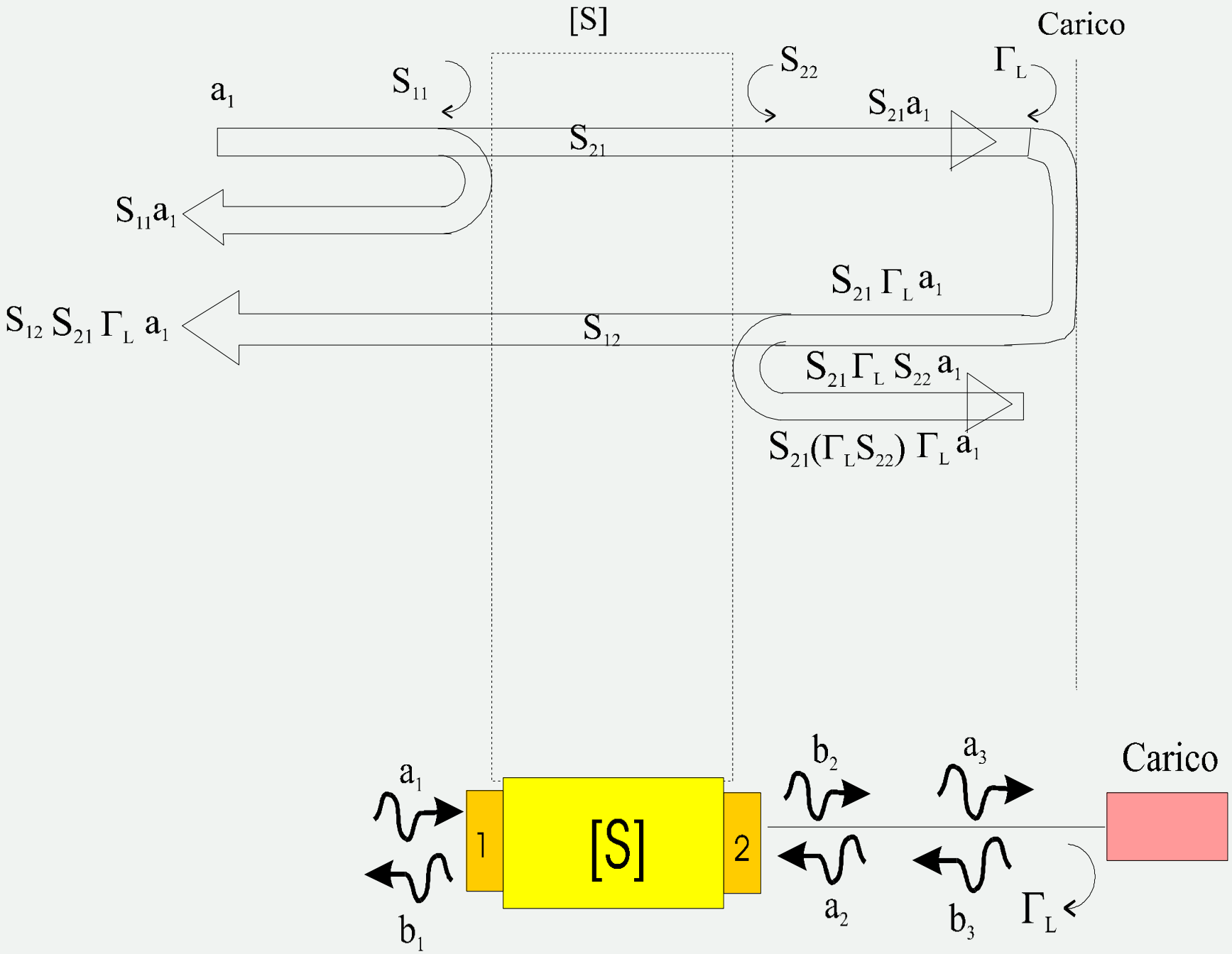

[S]
Carico
S_{22}
Γ_L
a_1
S_{11}
$S_{11}a_1$
a_1
b_2
a_3
Carico
1
[S]
2
b_1
a_2
b_3
Γ_L

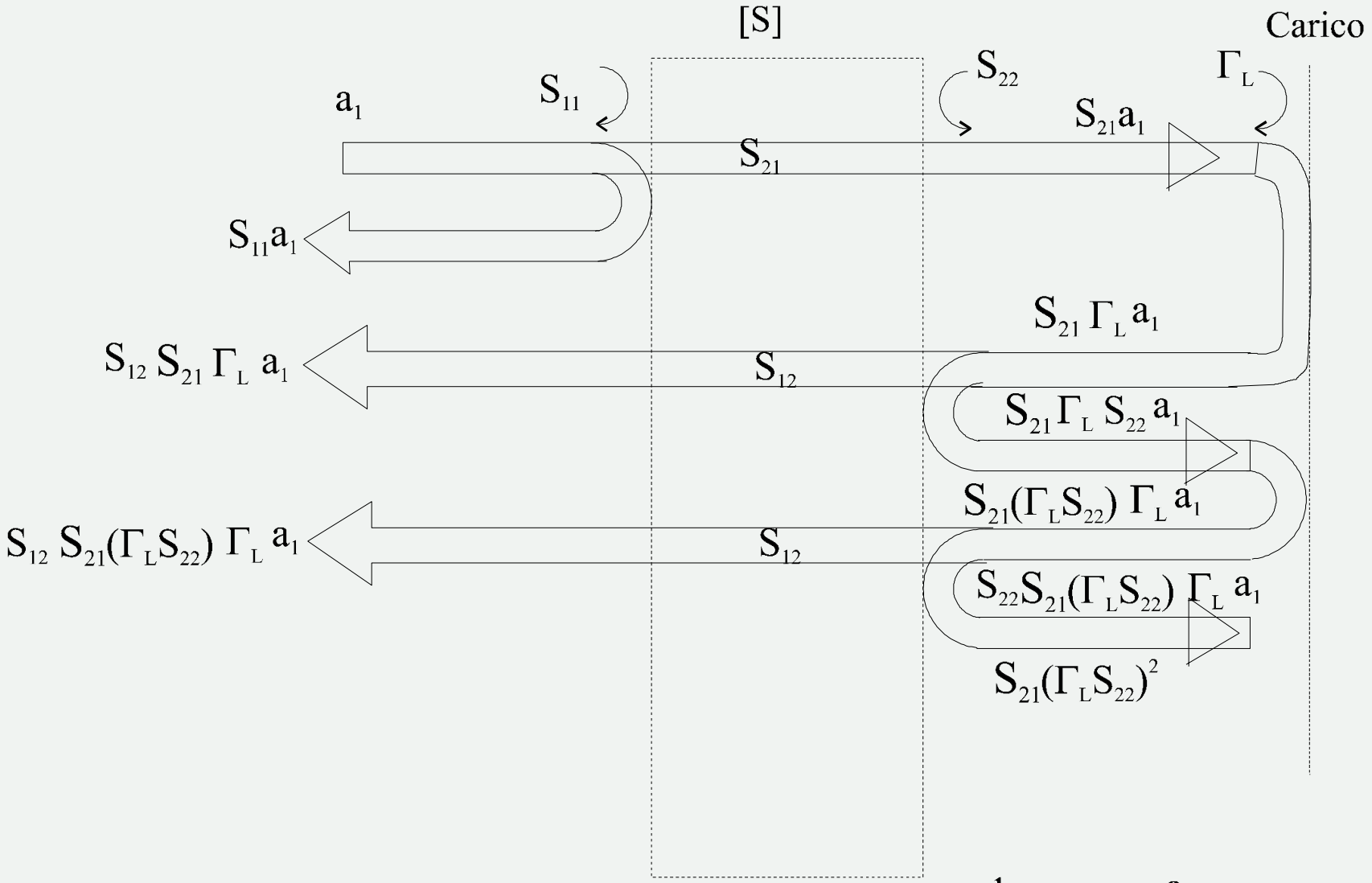

[S]
Carico
S_{11}
S_{22}
Γ_L
a_1
$S_{21}a_1$
S_{21}
$S_{11}a_1$
a_1
1
[S]
2
b_2
a_3
Carico
b_1
a_2
b_3
Γ_L

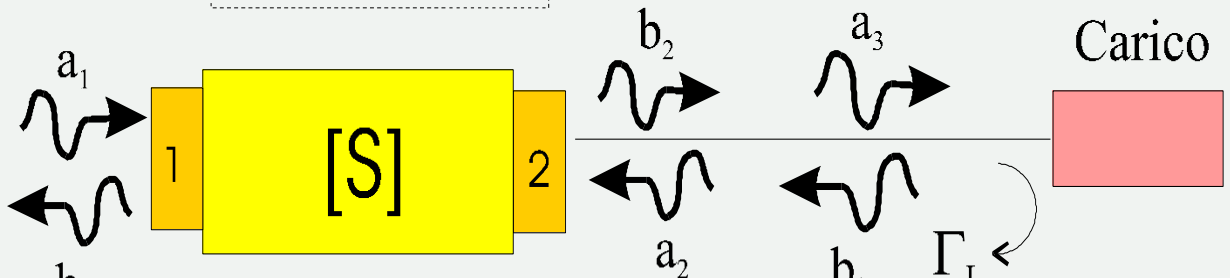

[S]
Carico
a_1
S_{11}
S_{22}
Γ_L
$S_{21}a_1$
S_{21}
$S_{11}a_1$
$S_{21}\Gamma_L a_1$
a_1
1
[S]
2
b_2
a_3
Carico
b_1
a_2
b_3
Γ_L

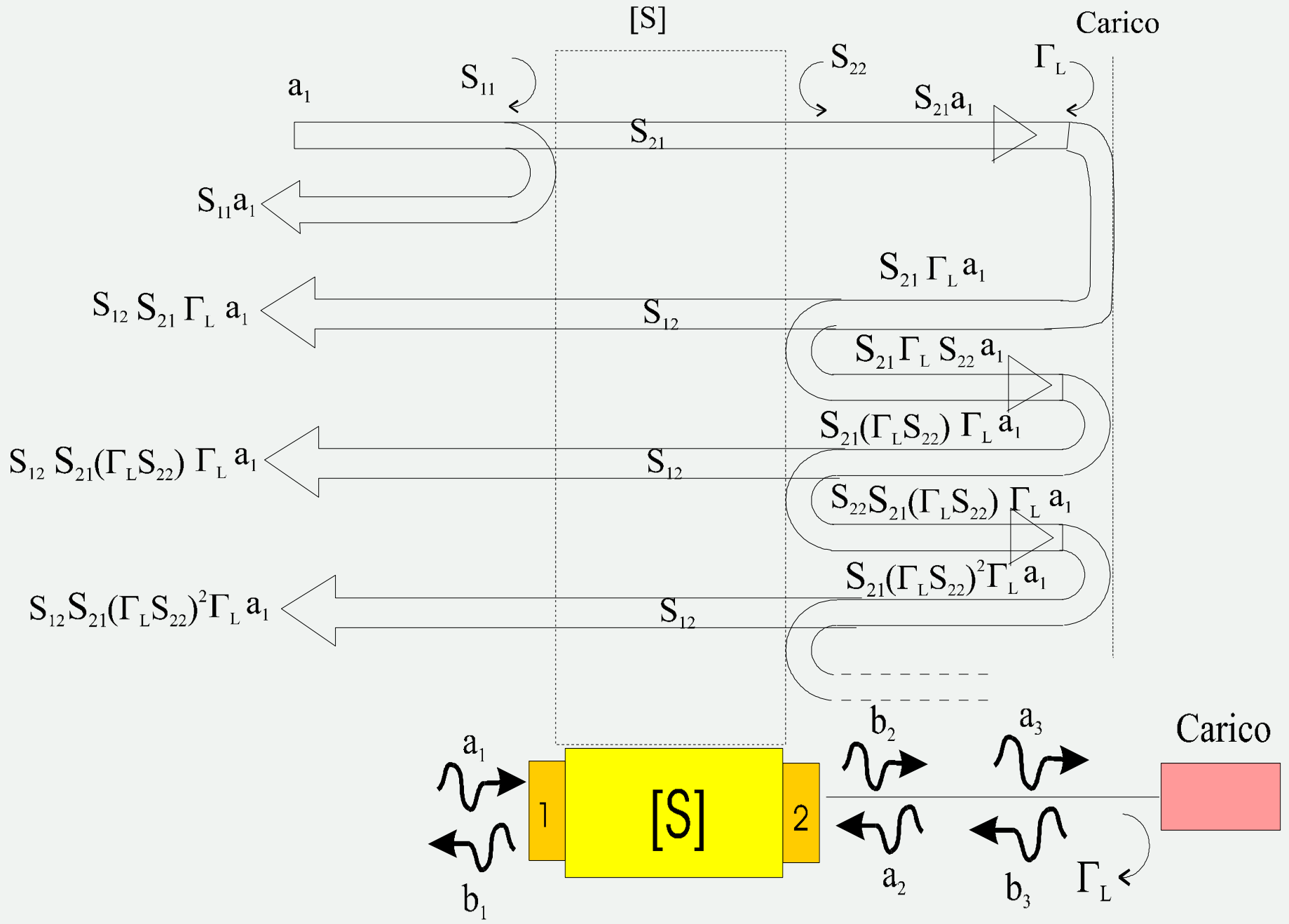

[S]
Carico
a_1
S_{11}
S_{22}
Γ_L
$S_{21}a_1$
S_{21}
$S_{11}a_1$
$S_{21}\,\Gamma_L\,a_1$
$S_{21}\,\Gamma_L\,S_{22}\,a_1$
$S_{21}(\Gamma_L S_{22})\,\Gamma_L\,a_1$

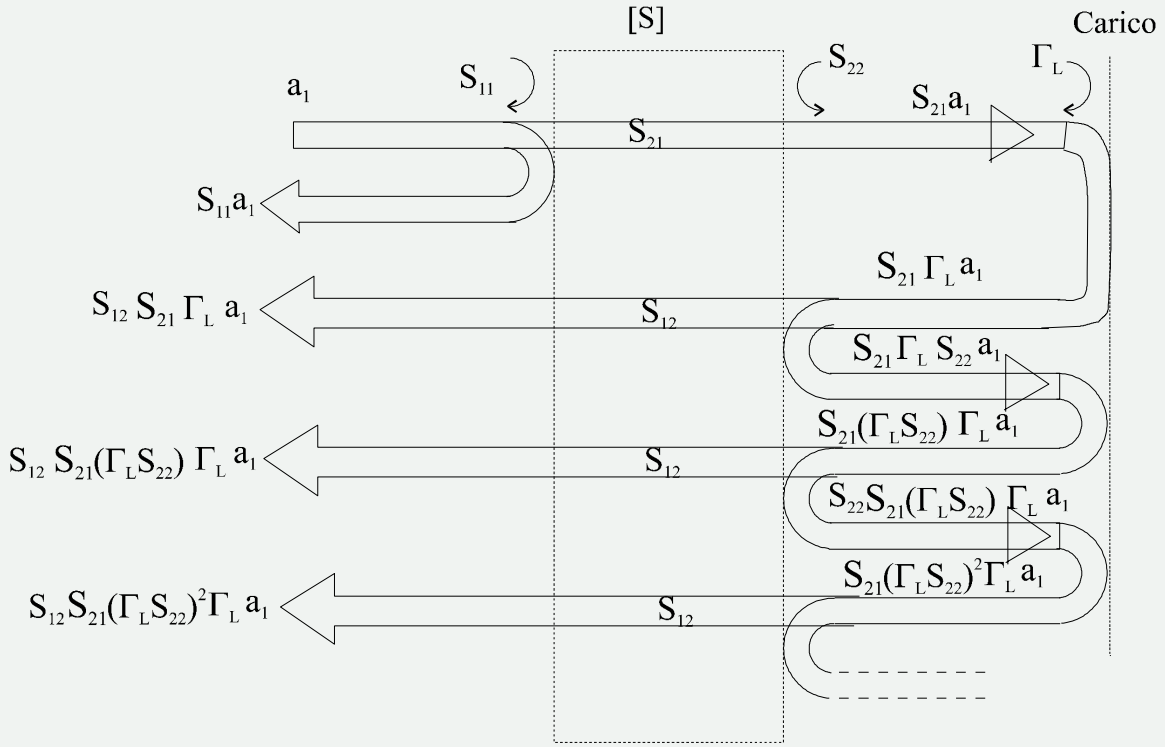

a_1
1
[S]
2
b_2
a_3
Carico
a_2
b_3
Γ_L


[S]
Carico
a_1
S_{11}
S_{22}
Γ_L
$S_{21}a_1$
S_{21}
$S_{11}a_1$
$S_{21}\,\Gamma_L\,a_1$
$S_{12}\,S_{21}\,\Gamma_L\,a_1$
S_{12}
$S_{21}\,\Gamma_L\,S_{22}\,a_1$
$S_{21}(\Gamma_L S_{22})\,\Gamma_L\,a_1$
a_1
1
[S]
2
b_2
a_3
Carico
a_2
b_3
Γ_L
b_1


[S]
Carico
a_1
S_{11}
S_{22}
Γ_L
$S_{21}a_1$
S_{21}
$S_{11}a_1$
$S_{21}\,\Gamma_L\,a_1$
$S_{12}\,S_{21}\,\Gamma_L\,a_1$
S_{12}
$S_{21}\,\Gamma_L\,S_{22}\,a_1$
$S_{21}(\Gamma_L S_{22})\,\Gamma_L\,a_1$
$S_{12}\,S_{21}(\Gamma_L S_{22})\,\Gamma_L\,a_1$
S_{12}
$S_{22}S_{21}(\Gamma_L S_{22})\,\Gamma_L\,a_1$
$S_{21}(\Gamma_L S_{22})^2$

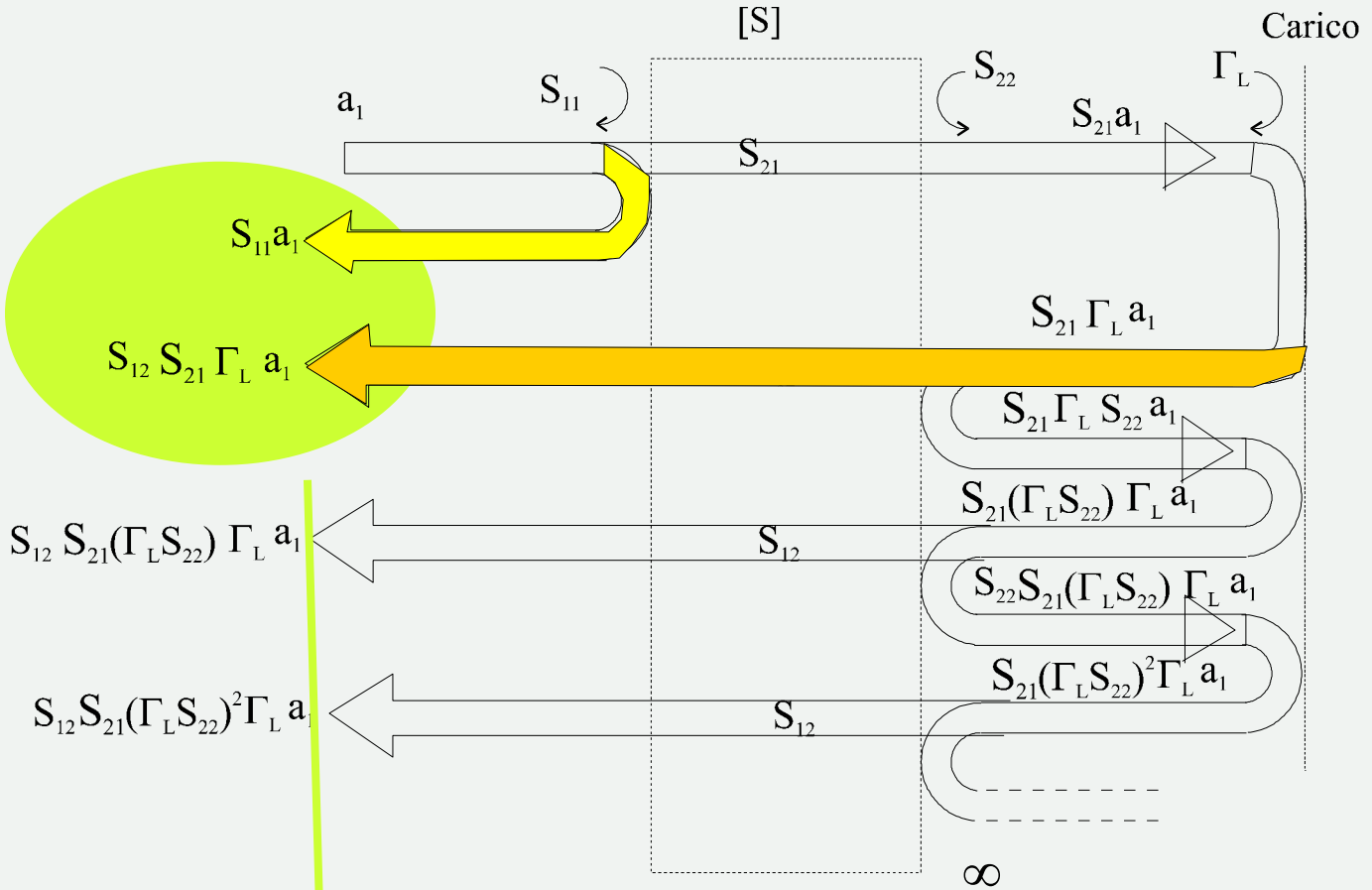

a_1
1
[S]
2
b_2
a_3
Carico
a_2
b_3
Γ_L


[S]
Carico
a_1
S_{11}
S_{22}
Γ_L
$S_{21}a_1$
S_{21}
$S_{11}a_1$
$S_{21}\,\Gamma_L\,a_1$
$S_{12}\,S_{21}\,\Gamma_L\,a_1$
S_{12}
$S_{21}\Gamma_L\,S_{22}\,a_1$
$S_{21}(\Gamma_L S_{22})\,\Gamma_L\,a_1$
$S_{12}\,S_{21}(\Gamma_L S_{22})\,\Gamma_L\,a_1$
S_{12}
$S_{22}S_{21}(\Gamma_L S_{22})\,\Gamma_L\,a_1$
$S_{21}(\Gamma_L S_{22})^2\Gamma_L\,a_1$
$S_{12}S_{21}(\Gamma_L S_{22})^2\Gamma_L\,a_1$
S_{12}
a_1
b_2
a_3
Carico
1
[S]
2
b_1
a_2
b_3
Γ_L



$$b_1 = S_{11}a_1 + S_{21}\Gamma_L S_{12}(1 + \sum_{n=1}^{\infty}(S_{22}\Gamma_L)^n)a_1$$

$$\frac{b_1}{a_1} = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L}$$



$$b_1 / a_1 = S_{11}a_1 + S_{21}\Gamma_L S_{12}(1 + \sum_{n=1}^{\infty}(S_{22}\Gamma_L)^n) \cong$$

$$\cong S_{11}a_1 + S_{21}\Gamma_L S_{12}$$

$S_{11} \rightarrow E_D$

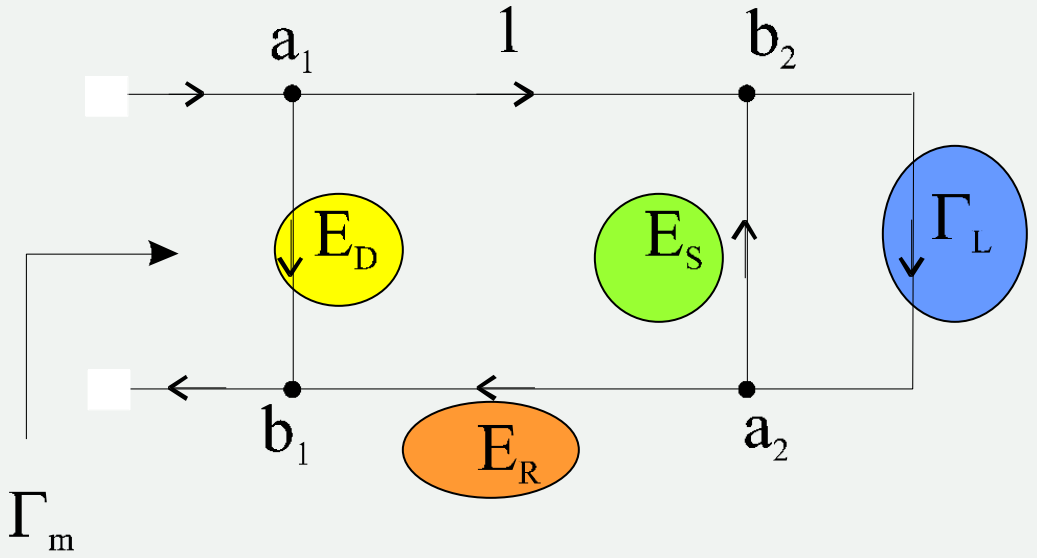
directivity error

$S_{12} \cdot S_{21} \rightarrow E_R$

reflection tracking error

$S_{22} \rightarrow E_S$

source match error

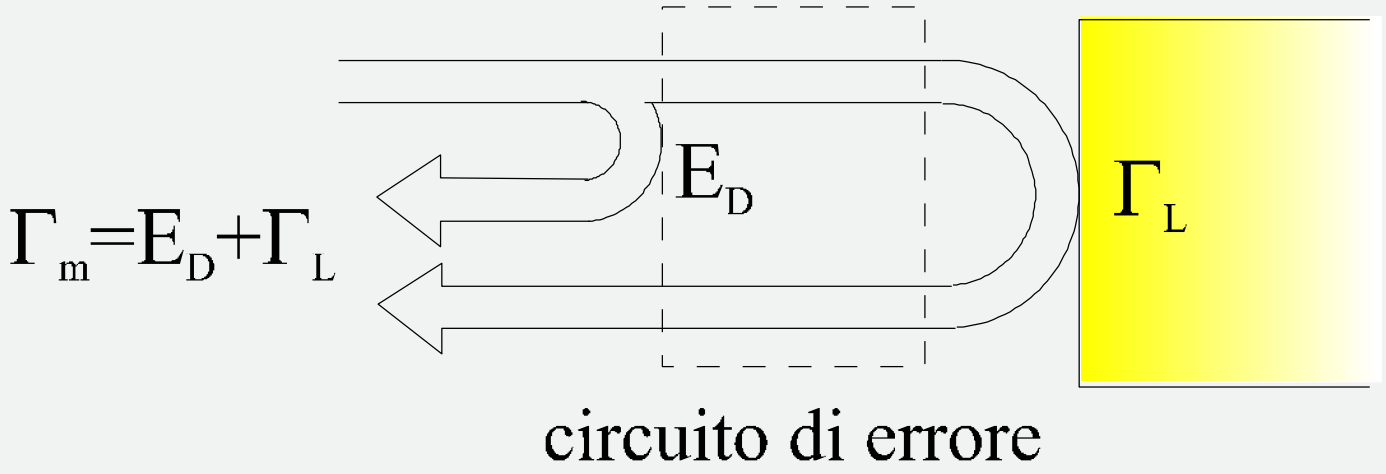


$$\Gamma_m = E_D + \frac{E_R \Gamma_L}{1 - E_S \Gamma_L}$$

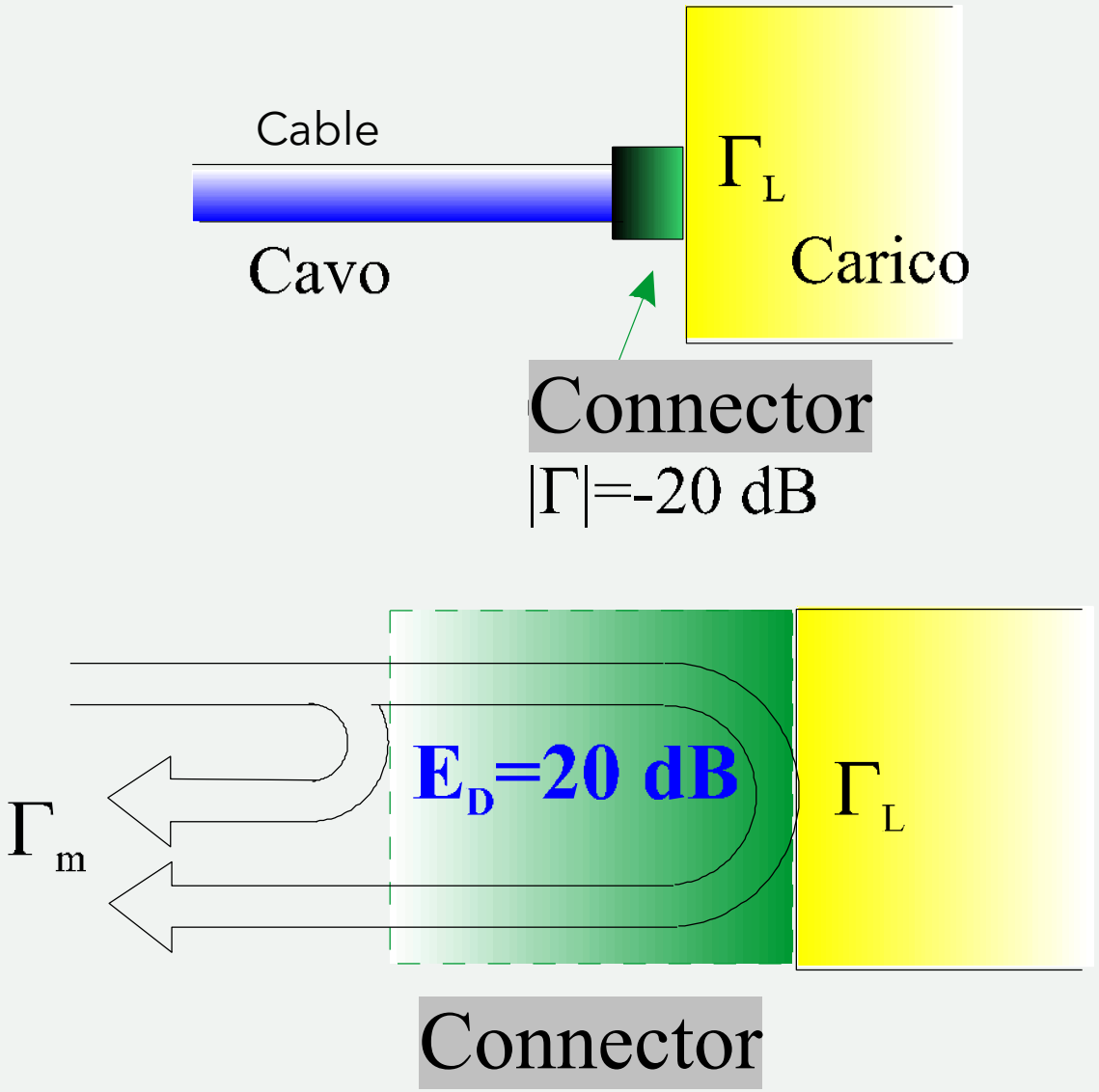
$$\Gamma_m \cong \Gamma_L + E_D + (E_R - 1)\Gamma_L + E_S \Gamma_L^2$$

Effective Directivity

$$\Gamma_m \cong \Gamma_L + E_D + (E_R - 1)\Gamma_L + E_S\Gamma_L^2$$



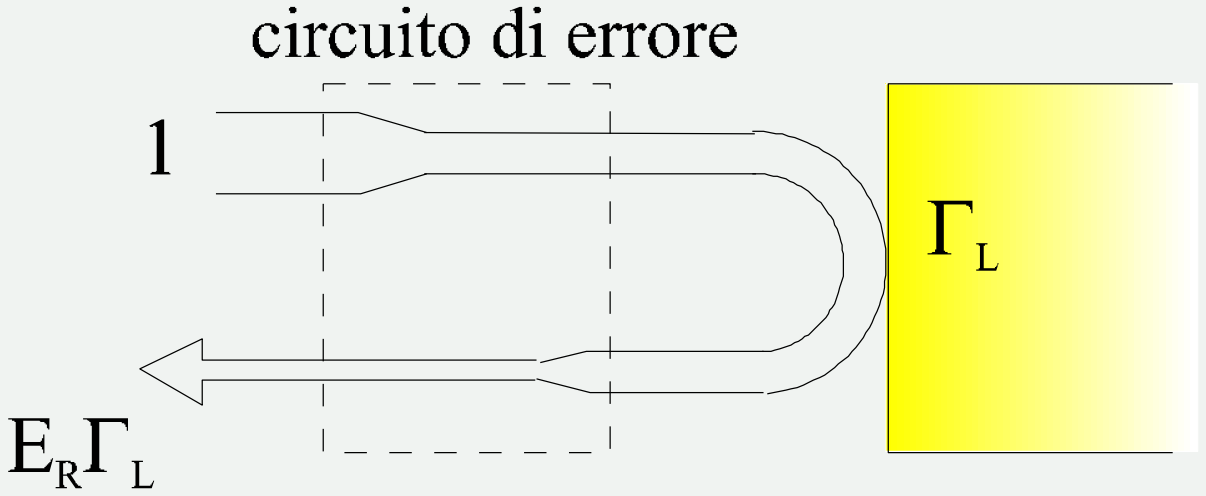
The importance of the connectors



The quality of the connectors limits the minimum measurable reflection coefficient

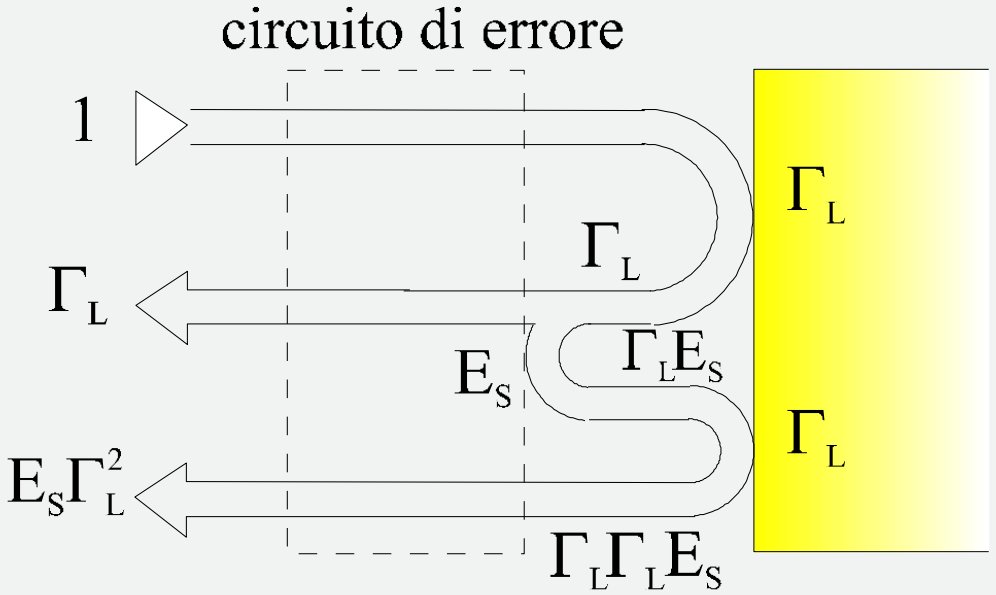
Effective reflection frequency response

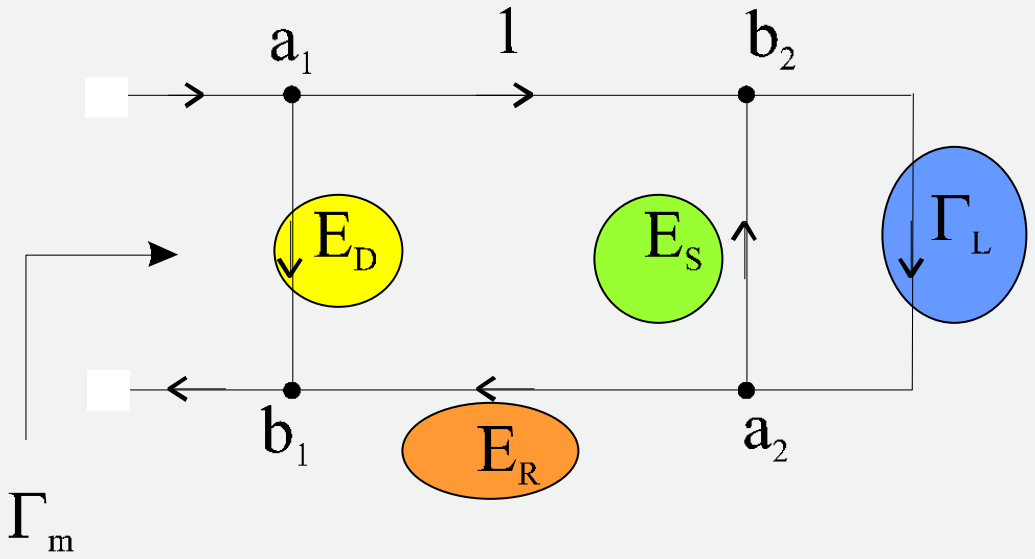
$$\Gamma_m \cong \Gamma_L + E_D + (E_R - 1)\Gamma_L + E_S\Gamma_L^2$$



Effective source mismatch

$$\Gamma_m \cong \Gamma_L + E_D + (E_R - 1)\Gamma_L + E_S\Gamma_L^2$$





$$\Gamma_m = E_D + \frac{E_R \Gamma_L}{1 - E_S \Gamma_L} \quad \Longleftrightarrow \quad \Gamma_L = \frac{\Gamma_m - E_D}{E_R + (\Gamma_m - E_D) E_S}$$

How we can evaluate E_D, E_R and E_S ?

…by measuring the reflection coefficient of three well-known and independent load impedances!

In the SOL calibration the load impedances we choose are a short circuit, an open circuit and a matched load

Short circuit

$$\Gamma_{mcc}=E_D-\frac{E_R}{1+E_S}$$

Open circuit

$$\Gamma_{mca}=E_D+\frac{E_R}{1-E_S}$$

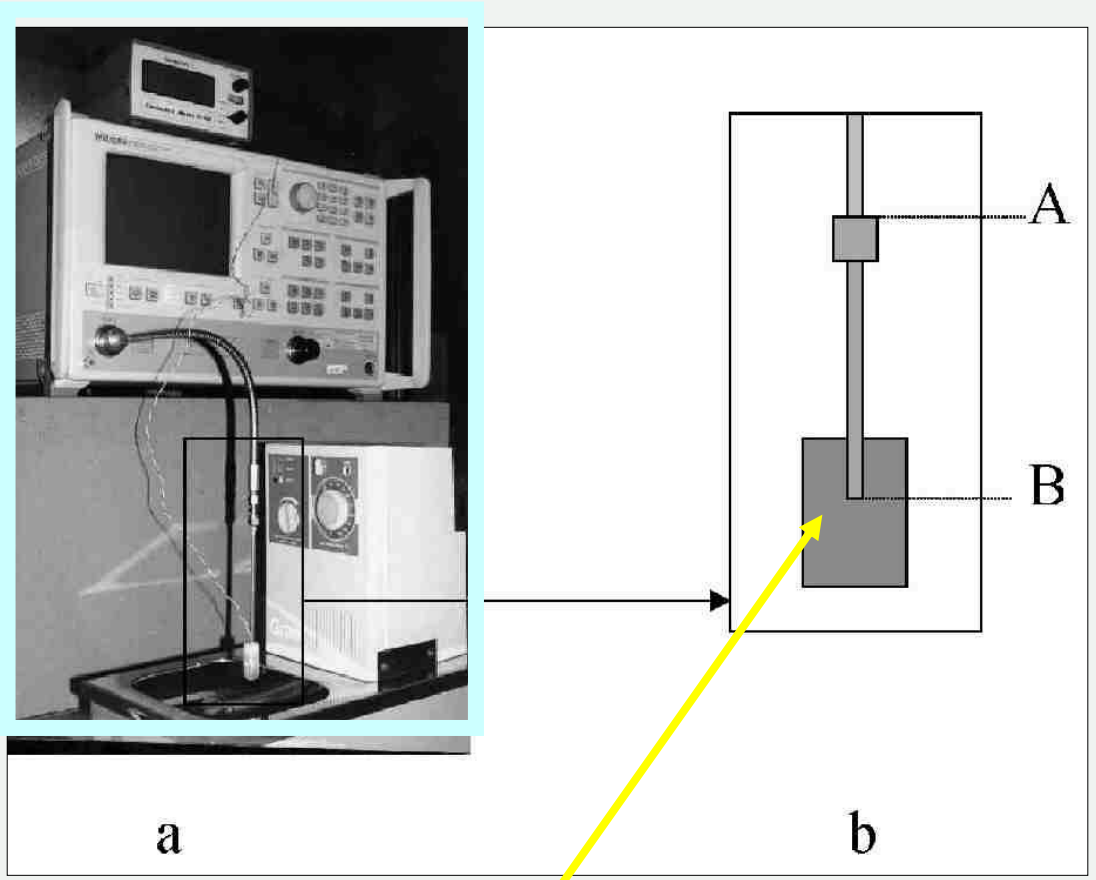
Matched load:

$$\Gamma_{mml}=E_D$$

$$\Gamma_L=\frac{\Gamma_m-E_D}{E_R+(\Gamma_m-E_D)E_S}$$

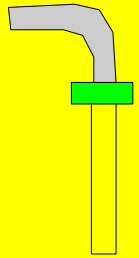
$$\begin{cases} E_D=\Gamma_{mml} \\ E_R=2(\Gamma_{mca}-\Gamma_{mml})(\Gamma_{mcc}-\Gamma_{mml})/(\Gamma_{mcc}-\Gamma_{mca}) \\ E_S=(2\Gamma_{mml}-\Gamma_{mcc}-\Gamma_{mca})/(\Gamma_{mcc}-\Gamma_{mca}) \end{cases}$$

Truncated coaxial cable



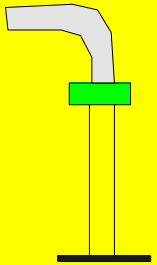
Desired data is at this section

Open Short Load calibration



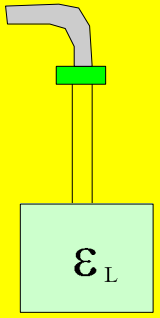
Open in air

$$\Gamma_{mk}^{O} = E_{Dk} + \frac{E_{Rk} - \Gamma_k^O}{1 - E_{Sk}\Gamma_k^O} \rightarrow E_{Dk}$$



Short circuit

$$\Gamma_{mk}^{S} = E_{Dk} + \frac{E_{Rk} - \Gamma_k^S}{1 - E_{Sk}\Gamma_k^S} \rightarrow E_{Rk}$$



Reference liquid

$$\Gamma_{mk}^{L} = E_{Dk} + \frac{E_{Rk} - \Gamma(\varepsilon = \varepsilon_L, f_k)}{1 - E_{Sk}\Gamma(\varepsilon = \varepsilon_L, f_k)} \rightarrow E_{Sk}$$

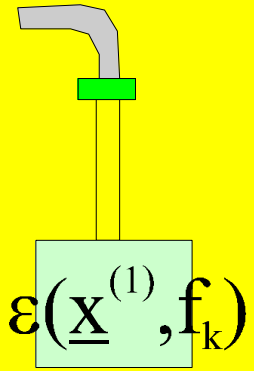
Accuracy depends on the referencve liquid

Partially known liquid: we can use the knowledeg of the dispersion law

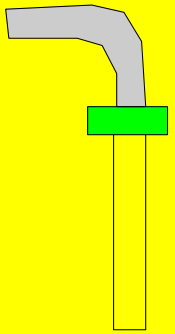
$$\varepsilon\left(\underline{x}, f\right) = \varepsilon_{\infty} + \frac{\varepsilon_0 - \varepsilon_{\infty}}{1 + \left(j2\pi f\tau\right)^{1-\alpha}}$$

$\underline{x} = (\varepsilon_{\infty}, \varepsilon_0, \tau, \alpha)$

First partially
Know liquid

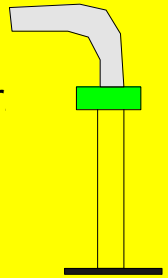


$$\Gamma_{mk}^{(1)} = E_{Dk} + \frac{E_{Rk} - \Gamma\left(\underline{x}^{(1)}, f_k\right)}{1 - E_{Sk}\Gamma\left(\underline{x}^{(1)}, f_k\right)}$$



Open in air

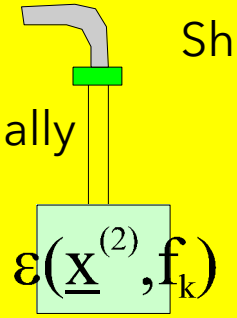
$$\Gamma_{mk}^{O} = E_{Dk} + \frac{E_{Rk} - \Gamma_k^O}{1 - E_{Sk}\Gamma_k^O}$$



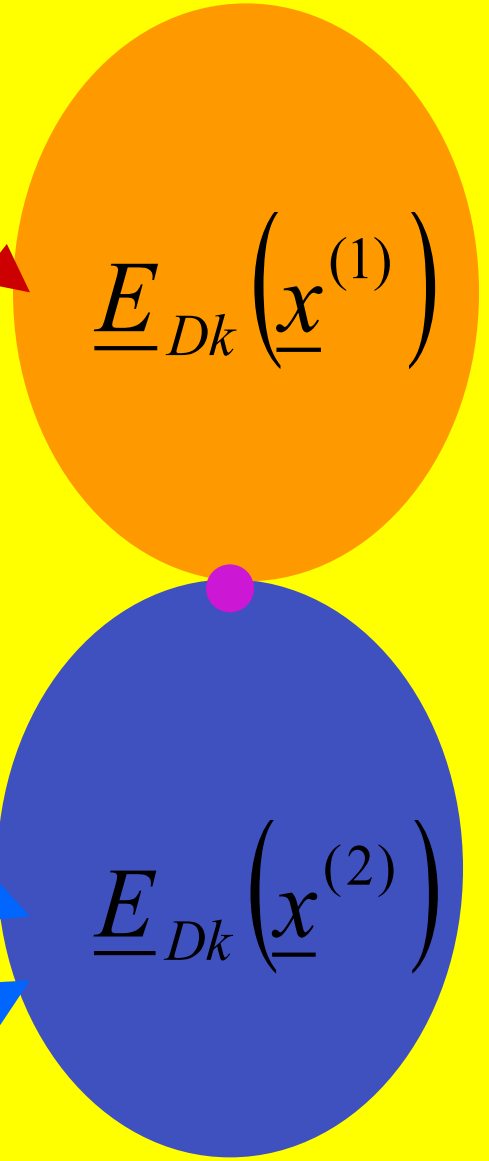
Short circuit

$$\Gamma_{mk}^{S} = E_{Dk} + \frac{E_{Rk} - \Gamma_k^S}{1 - E_{Sk}\Gamma_k^S}$$

Second partially
Know liquid



$$\Gamma_{mk}^{(2)} = E_{Dk} + \frac{E_{Rk} - \Gamma\left(\underline{x}^{(2)}, f_k\right)}{1 - E_{Sk}\Gamma\left(\underline{x}^{(2)}, f_k\right)}$$



Distilled water (T=26°)

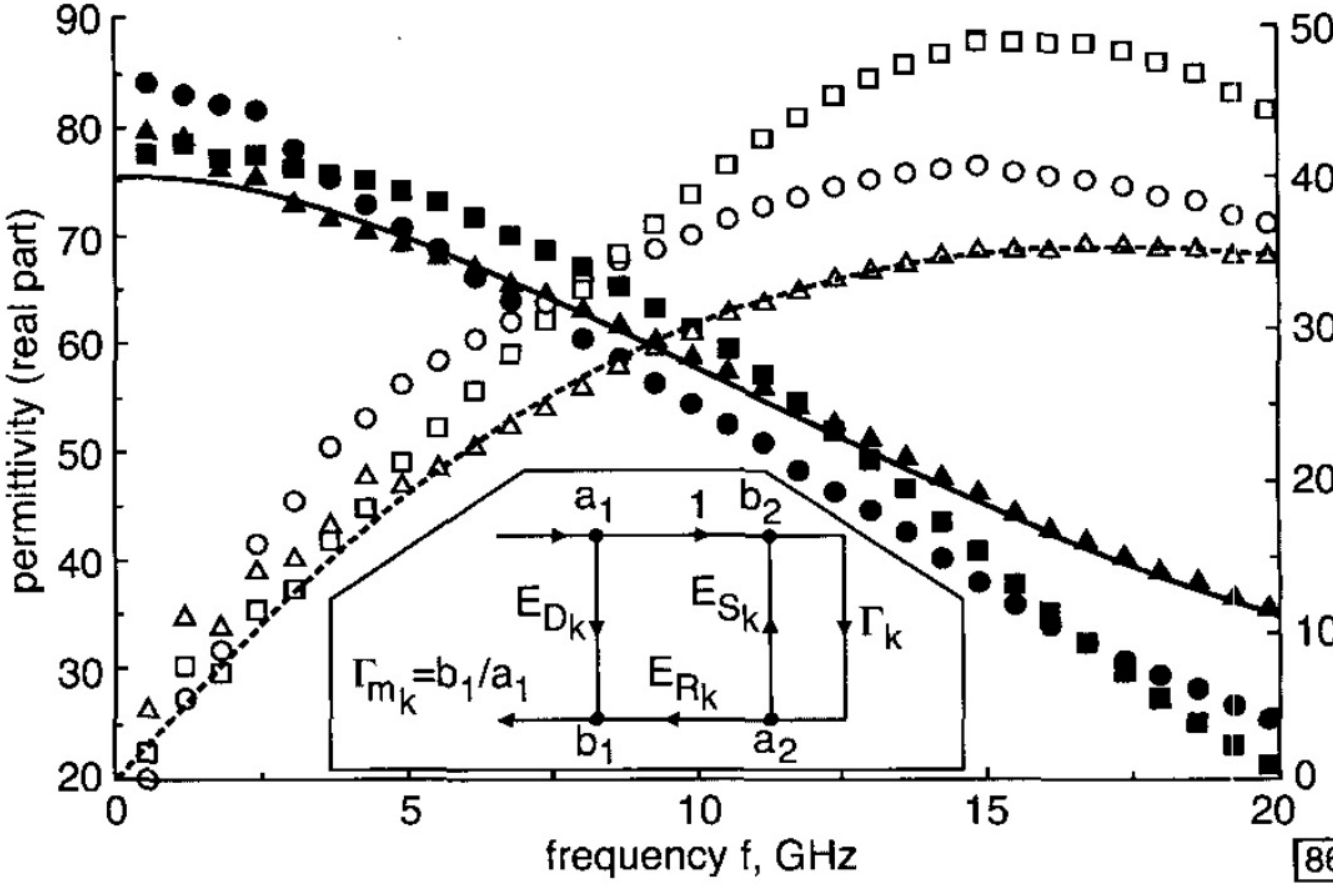


Fig. 1 *Relative permittivity of pure water*

■ ethanol, real part
● methanol, real part
▲ methanol and ethanol, real part
□ ethanol, imaginary part
○ methanol, imaginary part
△ methanol and ethanol, imaginary part
NBS data
——— real part
– – – – imaginary part

Characterization of polymers and composites

Polyethylenglycole

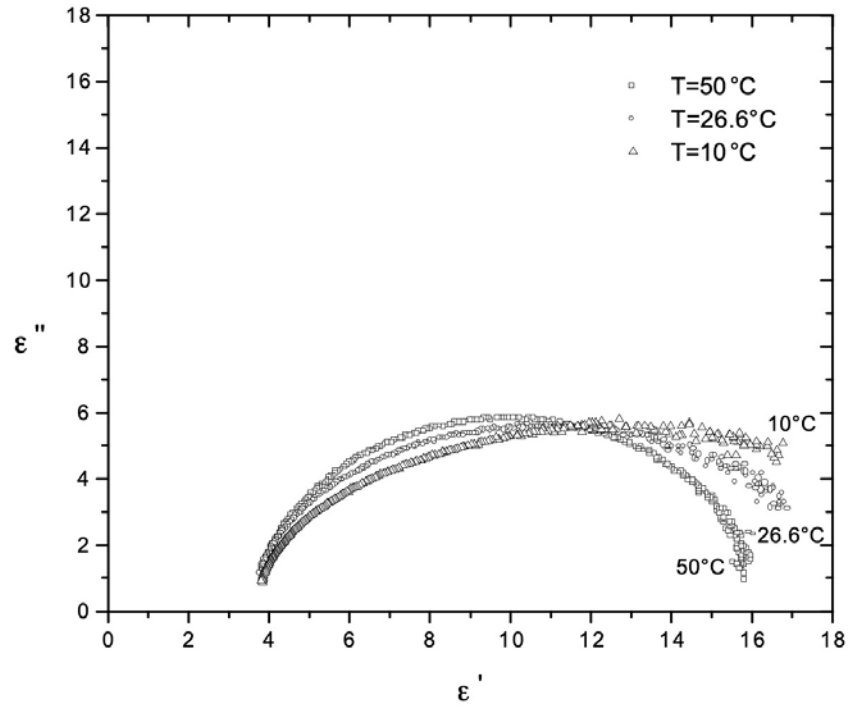
$OH - (CH_2 - CH_2 - O -)_n CH_2 - OH$

- Hevrilak-Negami model

$$\frac{\varepsilon(\omega) - \varepsilon_\infty}{\varepsilon_s - \varepsilon_\infty} = \left[1 + (i\omega\tau_0)^{1-\alpha}\right]^{-\beta}$$

where ε_∞ is the optical permittivity, ε_s is the static permittivity, τ_0 is a reference relaxation time and α and β are fit parameters, ranging from zero to one, describing the asymmetry and width of the relaxation time distribution. The Havriliak-Negami equation reduces to the Debye equation for $\alpha = 0$ and $\beta = 1$.

Hevrilak-Negami model



c)

Table 1

Polyethileneglycole Molecular weight	β	α	τ_o [ps]
200	0.98	0.07	67
400	0.99	0.05	58
600	0.99	0.05	53

M.W. 200 (10°C < T < 50°C)

$$\varepsilon_s = 21.54 - 0.1118 \cdot T$$

$$\varepsilon_\infty = 3.121 - 0.976 \cdot 10^{-2} T + 3.533 \cdot 10^{-4} T^2$$

$$\beta = 0.8874 + 0.262 \cdot 10^{-2} T$$

$$\alpha = 0.33 - 0.54 \cdot 10^{-2} T$$

$$\ln \tau_0 = -32.08 + 2786/T \ *$$

M.W. 400 (10°C < T < 50°C)

$$\varepsilon_s = 17.62 - 0.1079 \cdot T$$

$$\varepsilon_\infty = 3.357 - 1.36 \cdot 10^{-2} T + 2.928 \cdot 10^{-4} T^2$$

$$\beta = 0.881 + 0.279 \cdot 10^{-2} T$$

$$\alpha = 0.42 - 0.7 \cdot 10^{-2} T$$

$$\ln \tau_0 = -31.83 + 2662/T \ *$$

M.W. 600 (20°C < T < 50°C)

$$\varepsilon_s = 14.03 - 0.05838T$$

$$\varepsilon_\infty = 3.46 - 2.249 \cdot 10^{-2} T + 3.86 \cdot 10^{-4} T^2$$

$$\beta = 0.78 + 0.363 \cdot 10^{-2} T$$

$$\alpha = 0.35 - 0.684 \cdot 10^{-2} T$$

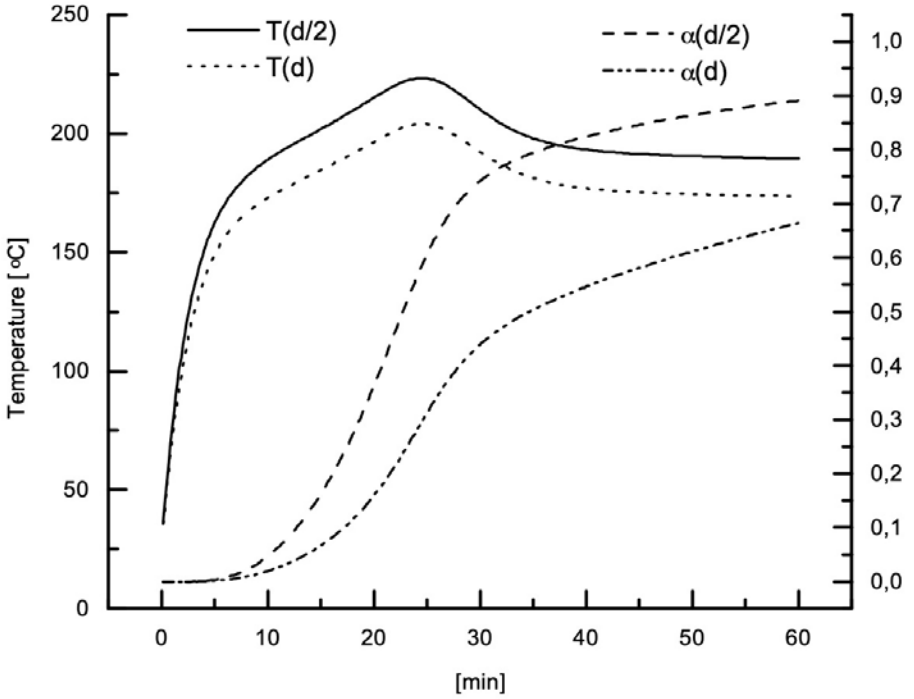
$$\ln \tau_0 = -30.94 + 2358/T \ *$$

* T expressed in Kelvin.

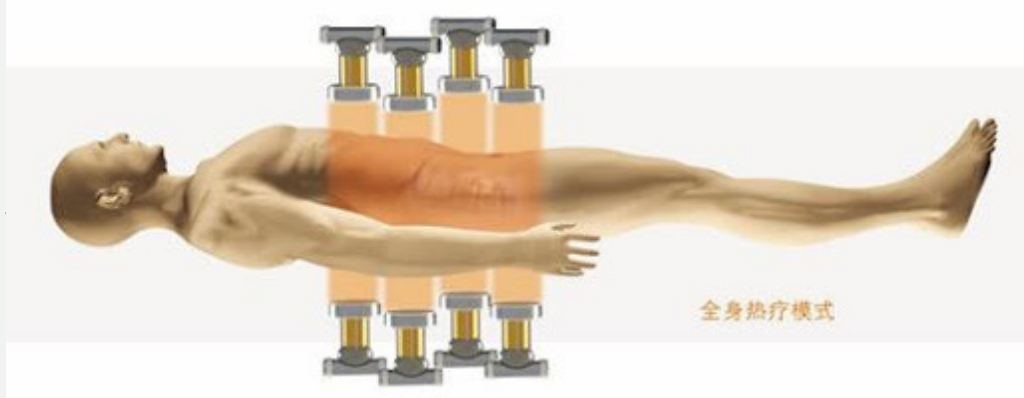
Numerical simulation of Microwave Cure of Epoxy Resins

$$\rho(T,\alpha)C(T,\alpha)\frac{\partial T(x,t)}{\partial t}=\frac{\partial}{\partial x}\left(K(T,\alpha)\frac{\partial T(x,t)}{\partial x}\right)+\rho(T,\alpha)H_T\frac{d\alpha}{dt}+p_{mw}(T,x,\alpha)$$

$$p_{mw}(T,x,\alpha)=\frac{\omega}{2}\varepsilon_0\varepsilon''(T,x,\alpha)\left|E(x)\right|^2$$



Hyperhemia:



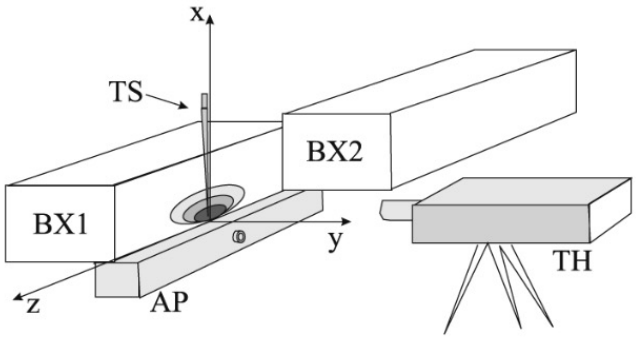


Fig. 5. Temperature measurement set-up: BX1,BX2: Plexiglas boxes filled by tissue phantom; AP. leaky-wave applicator; TH: thermocamera; TS: three-points thermocouple.

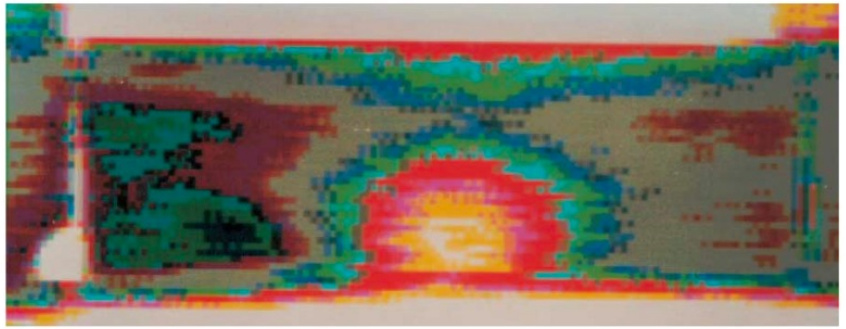


Fig. 6. Thermal image of the plane $y = 0$ of the phantom tissue acquired by the thermocamera.

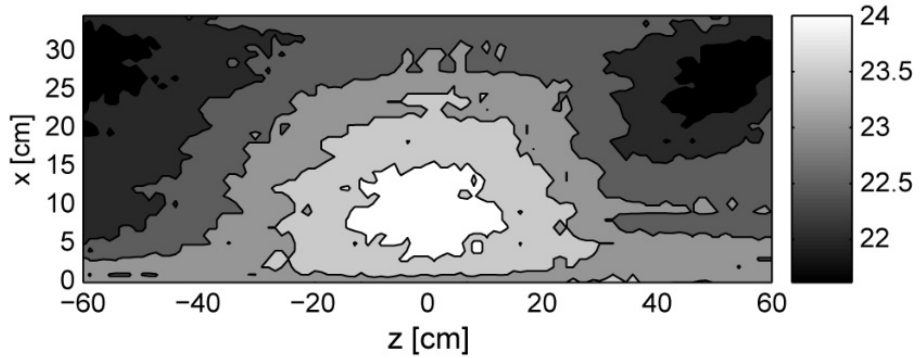
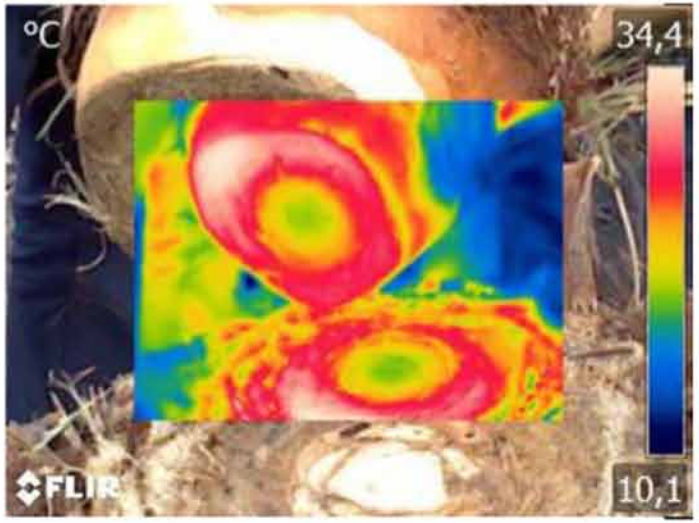


Fig. 7. Temperature distribution in the plane $y = 0$ of the tissue phantom after elaboration.

Applications to agriculture
Red Palm Weevil pest control

Microwave heating
Goal: kill the insect
do not kill the palm



Electromagnetic characterization of the materials involved

Fig. 1 Experimental set-up for permittivity measurements with the open ended coaxial cable technique.

Table 3 Electromagnetic properties of materials at 2.45 GHz

	Relative permittivity	Equivalent electric conductivity σ (S/m)	Penetration depth δ (cm)
Healthy palm wood (high density of fibres)	30.3	1.17	2.5
Healthy palm wood (low density of fibres)	46.0	1.40	2.6
Damaged tissue	50.7	2.93	1.3
Pine wood	3.1	0.068	13.4
Distilled water	73.6	1.29	3.5
Larva	35.3	1.04	3.1
Adult insect	9.3	0.38	4.3

Numerical siulation of the heating process in living plants

www.nature.com/scientificreports/

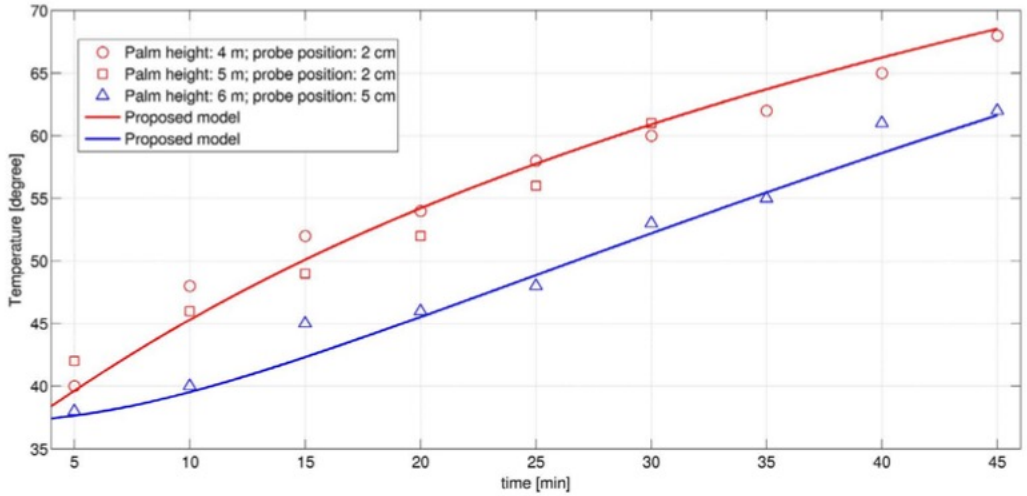
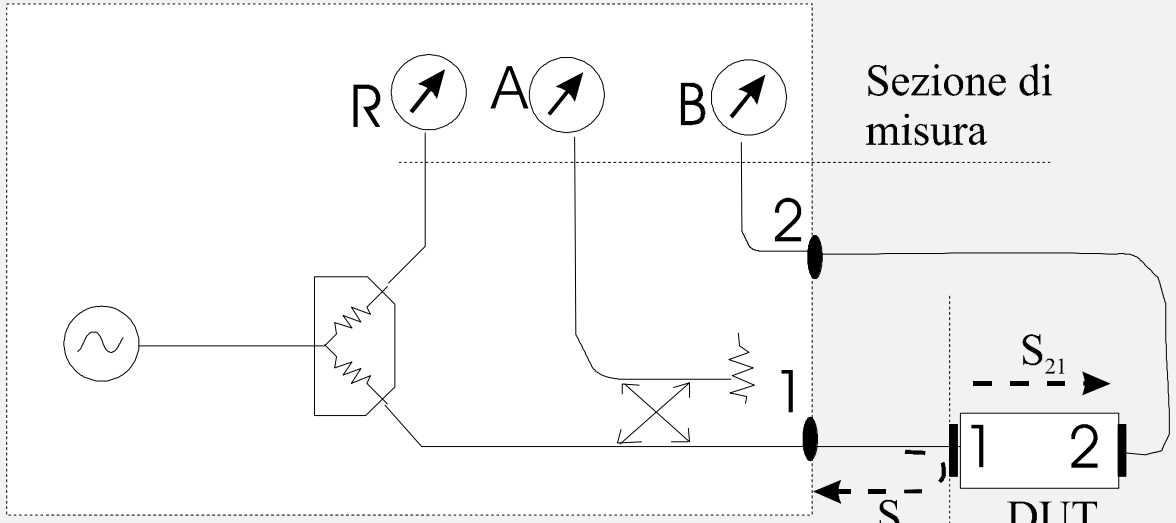
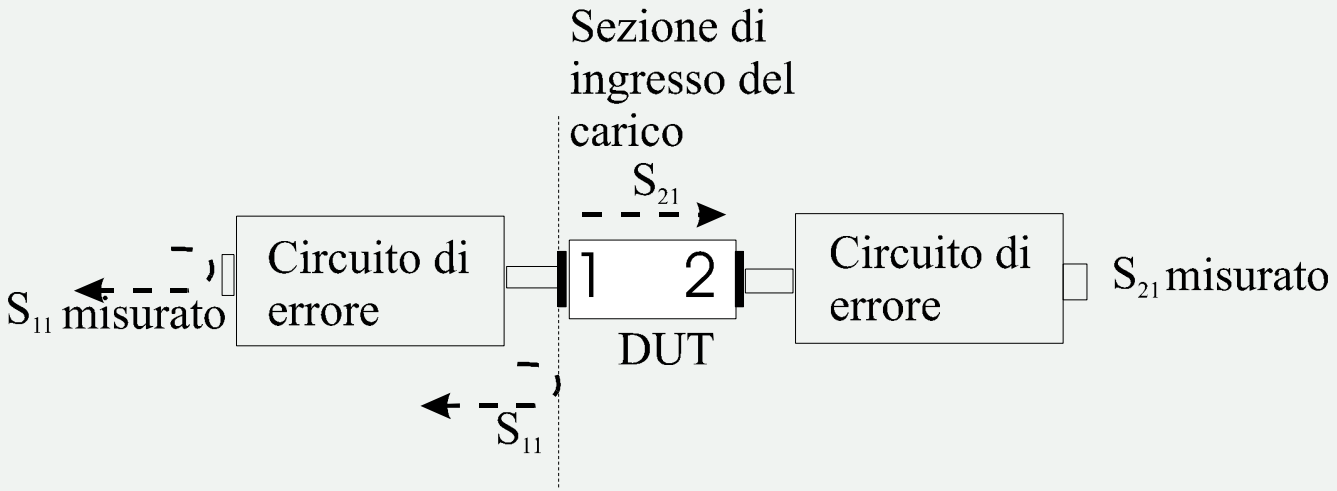


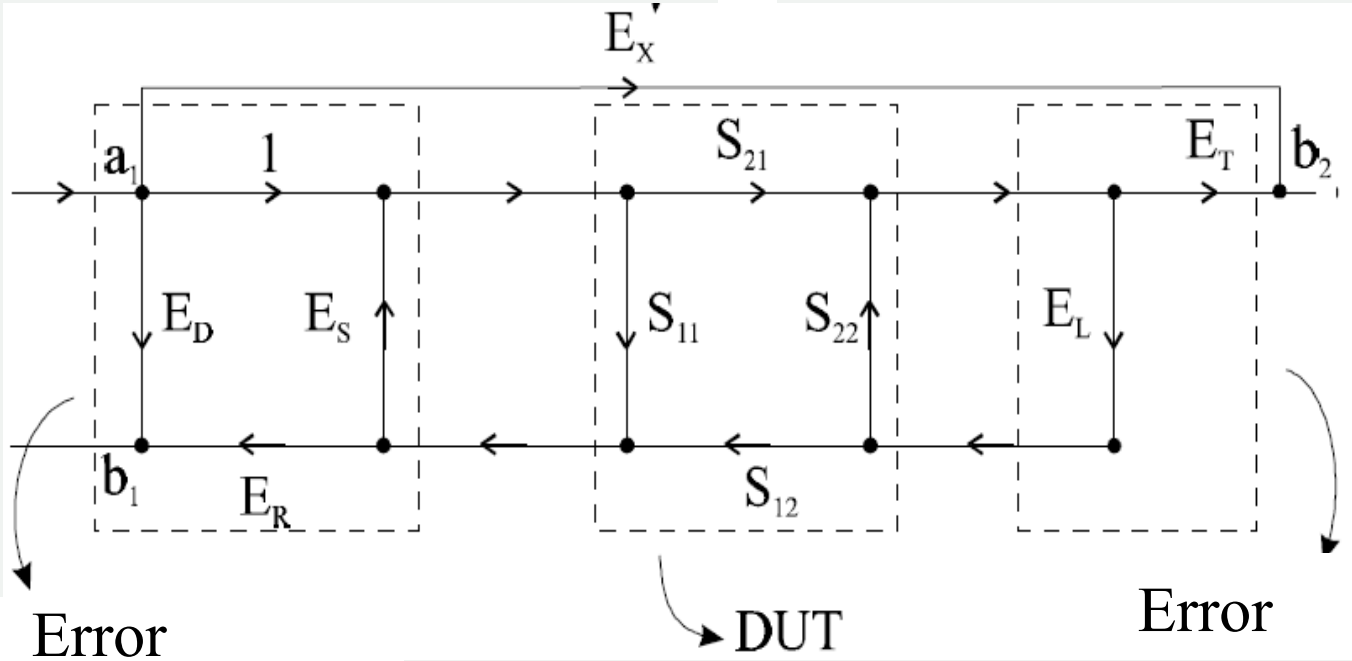
Figure 7. (**a**) Comparison of the time behavior of the temperature for three different palms with the proposed model; (**b**) one of the treated palms.

Errors in reflection and transmission measurements





Errors in reflection and transmission measurements



E_D: Directivity error

E_R: Reflection tracking error

E_S: Source match error

E_L: Load match error

E_T: Transmis. tracking error

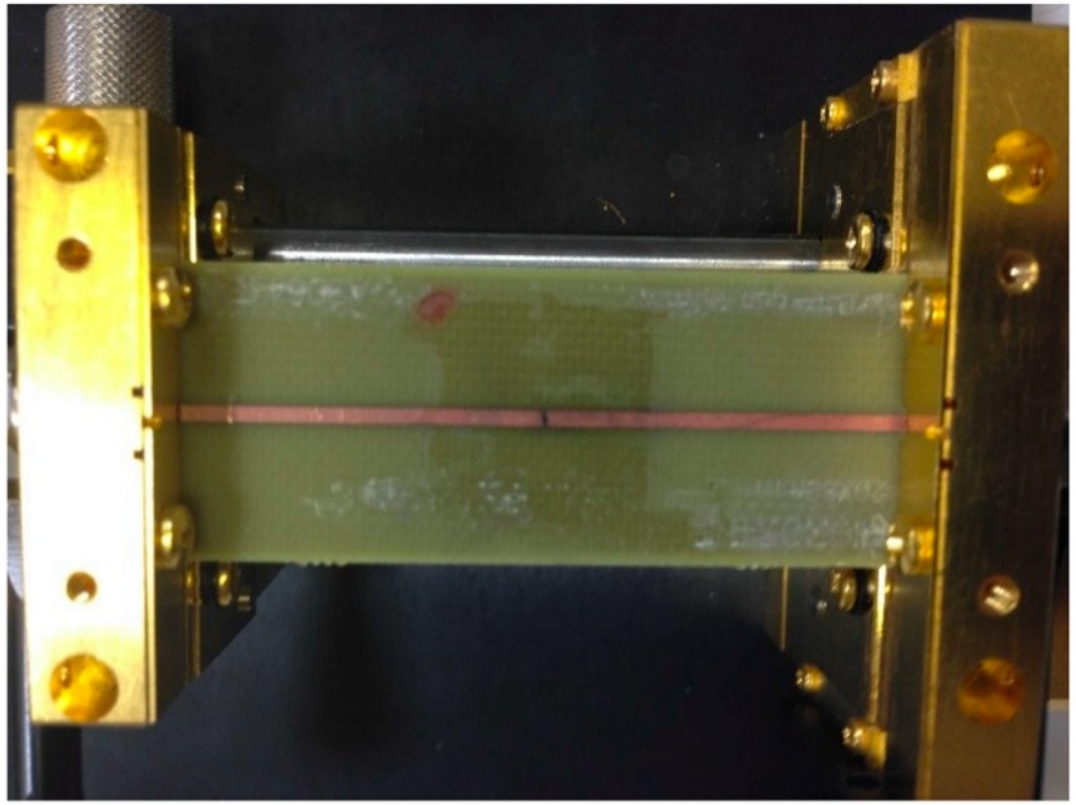
E_X: Isolation error

$$b_1 / a_1 \cong S_{11} + E_D + (E_R - 1)S_{11} + E_S S_{11}^2 + E_L S_{12} S_{21}$$

$$\frac{b_2}{a_1} \cong S_{21} + E_X + S_{21}(E_T - 1) + [E_S S_{11} + E_L S_{22}]S_{21}$$

Measurement using microstrips

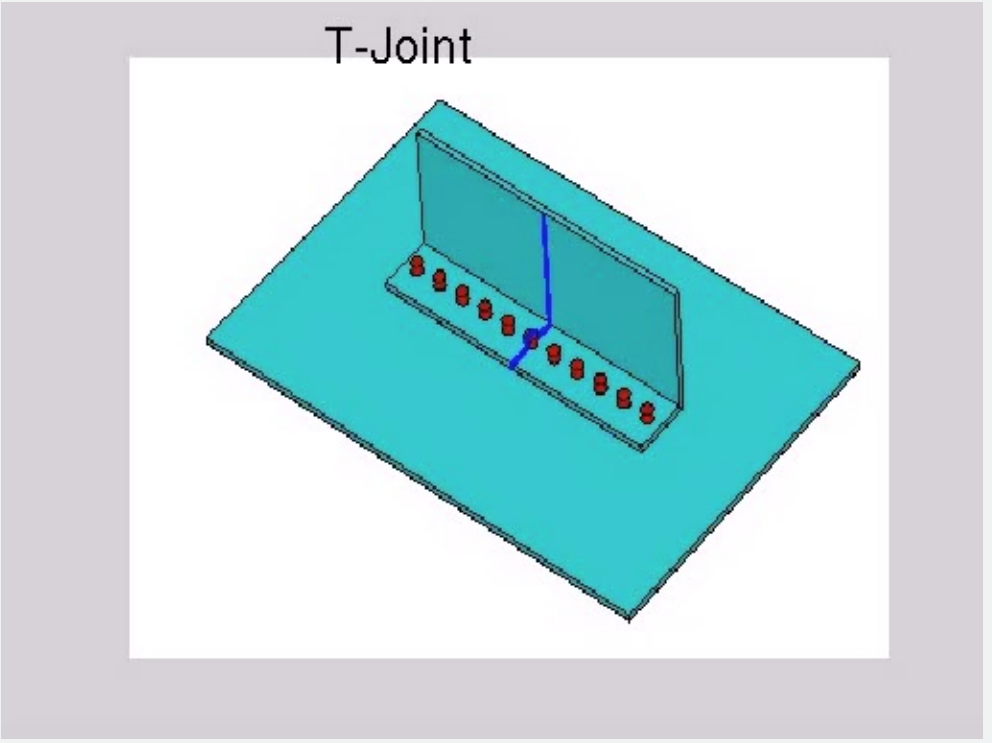
(a)



(b)

Figure 4. Setup for the microwave range characterization. (**a**) overall system; (**b**) details of the microstrip inserted into the text-fixture.

Applications



Questions ?